FVCC

PHYSICS OF TECHNOLOGY

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THE BINOCULARS

Waves, Physical Geometrical Optics

THE BINOCULARS

A Module on Waves, Physical Geometrical Optics

FVCC

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The Binoculars

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THE BINOCULARS

INTRODUCTION

This module will use binoculars to teach you certain principles of geometrical optics. You will not only learn about binoculars and telescopes, but you will also learn basic principles which will permit you to understand many other optical instruments and devices.

SPECIAL PREREQUISITES

This module is designed to follow *The Camera* module. Before starting work in this module, you will need certain skills and concepts. If you have studied *The Camera* module, you will have met these prerequisites. If you have not studied *The Camera* module, you may not understand parts of this module unless you have studied other modules or materials which have taught you to achieve essentially the same goals as those stated in *The Camera* module.

These prerequisites are related to the production of an image by a lens, various quantities associated with lenses, objects, and images, and how these quantities are related to one another.

In Section B of this module, you must be able to use a table of trigonometric functions to find the sine of an angle when you are given the angle or to find the angle when you are given the value of its sine.

GOALS FOR SECTION A

The following goals state what you should be able to do after you have completed this section of the module. These goals must be studied carefully as you proceed through the module and as you prepare for the post-test. The example which follows each goal is a test item which fits the goal. When you can correctly respond to any item like the one given, you will know that you have met that goal. Answers appear immediately following these goals.

1. *Goal:* Understand how the size or shape of the aperture of a telescope or monocular affects what you see through the instrument.

   Item: What happens to the image you see through a monocular when you cover half of the objective lens with your hand?

2. *Goal:* Know the difference between a virtual image and a real image.

   Item: Suppose you are given the parts of a telescope assembled on a bench without a tube or housing.

   a. What kind of image of an object is produced by the objective lens?
   b. How would you locate and show this image?
   c. What kind of image of the object is produced by the eyepiece?
   d. Could you locate and show this image in the same way as you did the image produced by the objective?

3. *Goal:* Know how to determine the magnification of a telescope or monocular.

   Item: Using a lab telescope, cathetometer telescope, monocular, or binoculars (either from your physics lab or elsewhere) use the even spaces of bricks of a neighboring building to determine the magnification of the instrument. You should be able to do this within three minutes.

4. *Goal:* Be able to identify the main parts of one of the optical instruments studied in this section and tell what each part does.

   Item: What are the main parts of a monocular and what does each part do?

5. *Goal:* Using principles of optics, know how to select the objective lens and eyepiece for a telescope and how to position these components to make a telescope which is in focus for a distant object.

   Item:

   a. From an assortment of five thin lenses, select and measure focal lengths until you have two lenses which can be used to construct a telescope with a magnification greater than one.
   b. Without looking through the eyepiece, position the two lenses on an optical bench so that a friend or your teacher sees a distant object in focus with no more than ± 1 cm adjustment in the position of the eyepiece. You should be able to do this within five minutes.

6. *Goal:* Know how to convert a telescope into a monocular and be able to determine the standard specification of this monocular.

   Item:

   a. Using a prism assembly supplied by your teacher, convert the telescope assembled in Item 5 into a monocular.
   b. Determine the standard specification of this monocular through direct measurements. You should be able to do this within five minutes.

7. *Goal:* Understand the relationship between the amount of light entering an optical instrument and the size of the aperture which admits the light.

   Item: Two monoculars are used to view the same object. One is a 6 X 30 monocular and the other is a 6 X 50 monocular. Which monocular has the most light in its exit pupil? How much more light is there in this exit pupil than that of the other monocular?

Answers to the Items
Accompanying the Preceding Goals

1. When you cover half of the objective lens with your hand, the amount of light entering the objective is cut in half. The image seen through the monocular will then look half as bright. (Note that covering part of an objective lens does *not* block the view of any part of what you are viewing.)

2. a. The objective lens produces a real, inverted image.
   b. You could locate and show this image by placing a white screen at its location. You would see the image on the screen.
   c. The eyepiece produces a virtual image. This image is upright compared to its object.
   d. You could not locate and show this image as you can for the objective lens. (Note that this virtual image is *not* the *real* image formed on your retina by your eye lens. The virtual image is actually the "object" for your eye lens.)

3. With one eye, look through the instrument at the brick wall. With the other eye look directly at the wall. You will see two images, one with each eye. Count how many brick spaces seen through the unaided eye fit into one brick space seen with the aided eye. This is the magnification.

4. The main parts are the objective lens, the prism system, and the eyepiece. The objective collects light from the object and forms a real image of the object. The prism system shortens the light path,

compared to a telescope, and inverts the image. The eyepiece magnifies the image of the objective lens to make the object look larger than it does with the unaided eye.

5. a. For a thin lens, you find the focal length by forming an image of a distant object on a screen. Then the focal length is the distance from the screen to the lens. In this way measure the focal lengths of the lenses.

   b. Position a longer focal length lens as the objective and a shorter focal length lens as the eyepiece. With these lenses spaced a distance apart equal to the value of $f_o + f_e$ the telescope should be nearly in focus for a distant object.

6. a. Place the prism assembly between the eyepiece and the objective, and align the assembly and the eyepiece to get a clear image of the object when viewing through the eyepiece.

   b. Using the method in Item 3, measure the magnification. Measure the diameter of the objective lens and express the result in mm. The magnification is the first number of the standard specification and the diameter is the second number.

7. The amount of light in the exit pupil is proportional to the square of the diameter. Therefore, there is $(50/30)^2$ times more light in the 6 X 50 monocular exit pupil than in the 6 X 30 monocular exit pupil. This result reduces to 25/9, or about 2.8 times as much light.

SECTION A

What Binoculars Do

INTRODUCTION

Binoculars are used to make objects appear larger than they appear to the unaided eye.

Simple magnifiers like the one in *The Camera* module were made by the ancient Romans. At first a simple lens was used just to burn things, as described by Pliny (50 A.D.), "If one exposes to the sun glass spheres filled with water, they produce so much heat that one can set clothes on fire with it." Although many lenses have been found from Graeco-Roman times, there is no firm evidence that such lenses were used as more than "a burning glass." Yet someone then may have observed their magnifying properties as well.

The first documented use of lenses for magnification was about 1300 A.D. Shortly afterward, spectacles began to be used in several countries. Some time around 1600 the telescope was invented, probably in the country of Holland.

Galileo used a description of this invention and later constructed several telescopes, one of which he used to make a series of famous observations of the earth's moon, and of Jupiter and its moons. His interpretations of these observations set off a major controversy which eventually resulted in his conviction for heresy by The Inquisition. Yet Galileo was correct in both his observations and interpretation.

You will learn about the telescope in this module, and you will see how a monocular is made from a telescope. A binoculars is simply two monoculars, one for each eye.

EXPERIMENT A-1. The Monocular

You have been provided a monocular for this experiment. We wish to investigate this optical system.

You should now take out the work sheets at the end of this module. Write answers to questions, and complete the tables on those sheets as you do the experiment.

1. Look through the *eyepiece* (small end) of the monocular at some scene in the room. Describe the scene. Do things look smaller or larger? Is everything upside down? Describe what you can about the nature of the scene in the small end of the monocular.

2. Now look through the *objective* (large end) at the same scene and describe it as you did in step 1.

3. Mount the monocular on the optical bench with the adjustable diaphragm in front of the objective lens (larger end) as shown in Figure 1. Look through the eyepiece (smaller end) toward a well-lighted area. As you look through the eyepiece, open and close the diaphragm. Describe what happens to the scene as the diaphragm is opened and closed.

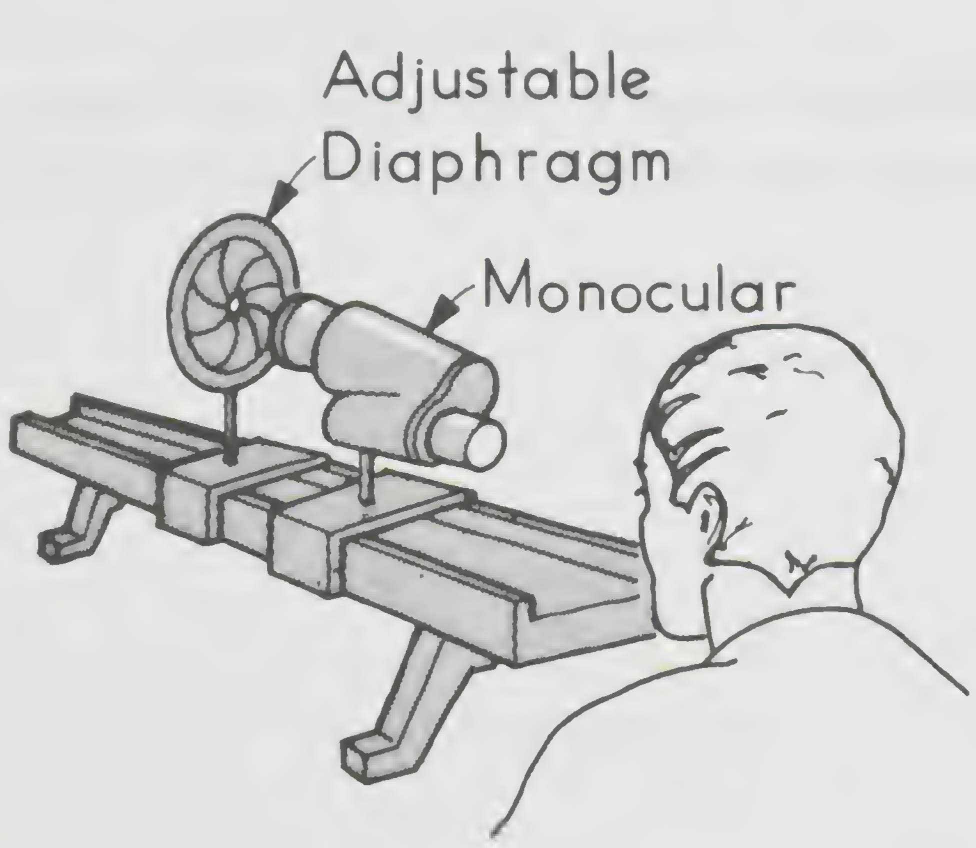

Figure 1.

4. Is the field of view (the area which you are able to see while looking through the monocular) reduced as the diaphragm is made smaller?

5. With your monocular on the optical bench, place a ground glass screen behind the eyepiece as shown in Figure 2. Can you obtain a clear picture of a distant scene on the screen by moving it toward or away from the eyepiece?

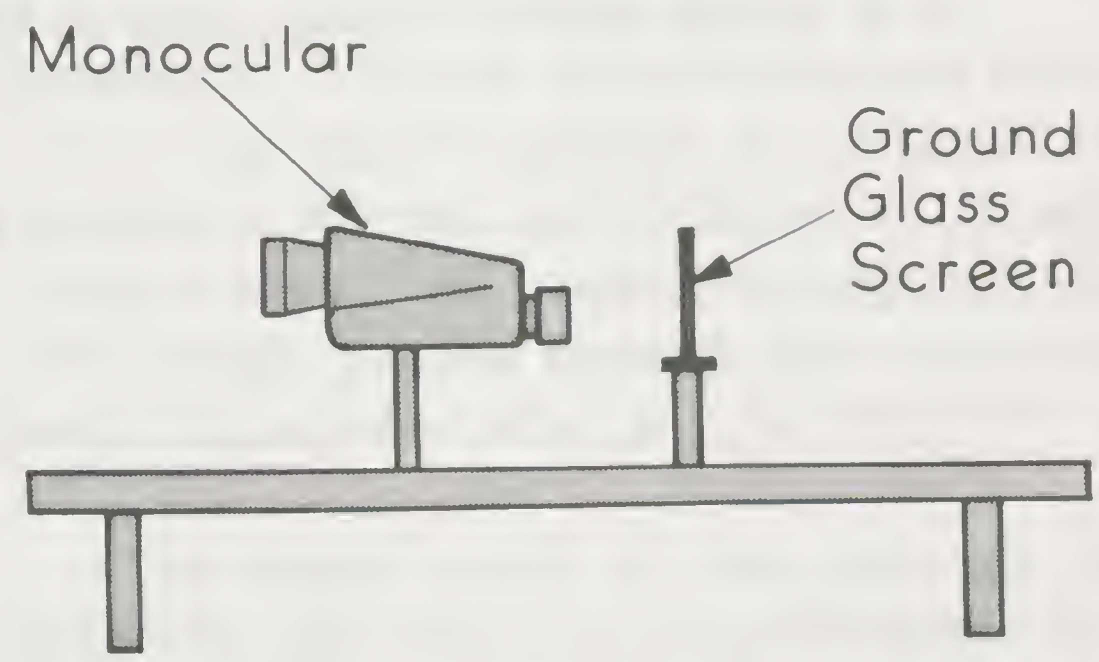

Figure 2.

6. Why do you think a clear scene cannot be formed on the screen?

7. As you move the screen in and out, you should see a circle of light formed on the screen. Adjust the screen until you have the most clearly defined circle. As you observe the circle of light on the screen, touch the tip of your pencil to the front of the objective lens. Does this produce any change in the circle of light on the screen? Describe any change you observe. What is the circle of light an image of?

8. Somewhere on the monocular you will find a combination of two numbers, such as 7 X 35 or 6 X 50. What are the numbers on your monocular?

9. Measure the diameter (D) of the objective lens in inches (in), centimeters (cm), and millimeters (mm).

10. Do any of these diameter measurements come close to either number on your monocular? Which one?

11. Recalling the effect of the size of the diaphragm on the monocular scene in an earlier part of this experiment, what do you think is the significance of the diameter? What effect does the diameter of the objective lens have on the image?

12. With one eye look through the monocular at evenly spaced objects, such as the lines on a brick wall or concrete block wall. With both eyes open, try to get the scenes seen through each eye to line up together. (You may have to hold the monocular in a clamp.) Count how many units (bricks) seen with the naked eye lie inside of one unit seen through the monocular. This number is the *magnification.*

13. Does this magnification correspond approximately with either number on the monocular? If so, which one?

THE OBJECTIVE LENS AND THE EXIT PUPIL

The objective lens of the monocular is the component of the instrument which lets in and limits the light from the scene that is viewed. In this type of optical device, the objective lens is also known as the *entrance pupil.* You have seen that an image of the objective lens is formed closely behind the eyepiece. This final image of the entrance pupil is called the *exit pupil.* The location of the exit pupil is the exact spot where the pupil of your eye should be placed in order to properly see through the monocular. In fact, if any optical instrument is used with the monocular, its entrance pupil should be positioned at this exit pupil.

Question 1. Although it is not a very practical idea, suppose another monocular were used to look through the first monocular in order to obtain more magnification. Where should it be placed behind the first?

THE IMAGE AND MAGNIFICATION

The image of the entrance pupil can be seen on a screen outside the monocular. But the image of a distant scene, which you see by looking through the instrument, could not be found on a screen outside the instrument. Still you could make some observations about the image. For one thing, it looks bigger than the distant scene looks when viewed through the monocular backwards (with the eyepiece toward the distant scene). One specification which is usually printed somewhere on the monocular or binoculars is a number that tells how much larger the scene looks through the instrument than it does when viewed directly. This number is known as the *magnifying power* or *magnification*, and it is typically from six to ten. Technically, it should be called *angular magnification.* A seven-power monocular makes distant objects appear seven times as large.

Question 2. Suppose two seven-power monoculars were placed eyepiece to eyepiece and you looked through one objective lens toward a distant scene. What would the apparent size of the image be seen through the two instruments compared to what is seen directly?

The image which is seen through the monocular and which cannot be focused on a screen is called a *virtual* image. It is different from an image like the exit pupil or the image on the film of a camera, both of which can be focused on a screen. These are called *real* images.

To gain a greater understanding of how the monocular works, it would be helpful to have its components outside of any housing, so that they can be studied separately, then together as a monocular. Then you could also look for images inside the monocular. You can perform this study by doing the next experiment.

EXPERIMENT A-2. Monocular Components

In this experiment we wish to examine the main components of the monocular. There are two lenses (an objective and an eyepiece) and a prism in the monocular. First look at the two lenses.

Take out the work sheets at the end of this module. Write answers to questions and complete the tables on these sheets as you do the experiment.

1. Use a distant object and the optical bench to measure the focal length in centimeters of each lens. (As in *The Camera* module, the focal length, f, is the distance from the lens to a well-focused real image of a distant object.)

2. Now place both lenses in lens holders on the optical bench. Look through both lenses toward a distant object. Adjust the distance between the lenses until a clear image is seen. This is a simple telescope. What is the distance between the lenses in centimeters?

3. How is this distance related to the focal lengths of the lenses?

4. Which lens, lens 1 or lens 2, are you using for the objective and which for the eyepiece?

5. Describe the qualities of the image.

6. Estimate the magnification by the method you used in Experiment A-1.

7. Turn the optical bench around and look through the lenses so that the eyepiece and objective are interchanged. Now which lens is the objective and which is the eyepiece?

8. Now describe the qualities of the image.

9. Estimate the magnification.

10. With the optical bench turned so that the lens of longer focal length is nearest the object, place a ground glass screen on the optical bench behind the eyepiece as shown in Figure 3. Try to focus a clear image of your distant object on the screen. Make sure any images formed are produced by both lenses. If placing your hand in front of the objective does *not* block out the image, it is formed only by the eyepiece. Can you obtain an image?

11. Place the ground glass between the lenses and attempt to obtain a clear image. How far from the objective is such an image formed?

12. Is this distance related to the focal length of either lens?

Place the prism system in a holder on the optical bench. The entry to the prism should be between the real image found in step 11 and the objective. As shown in Figure 4, make sure the entry to the prism is on the axis of the objective and the exit is on the axis of the eyepiece. Adjust the eyepiece, and prism if necessary, to obtain a clear image when viewed through the eyepiece. You now have a monocular.

13. Describe the image.

14. How is the image different from that of the simple telescope?

15. What is the distance between the lenses?

16. How does this compare with the distance in step 2?

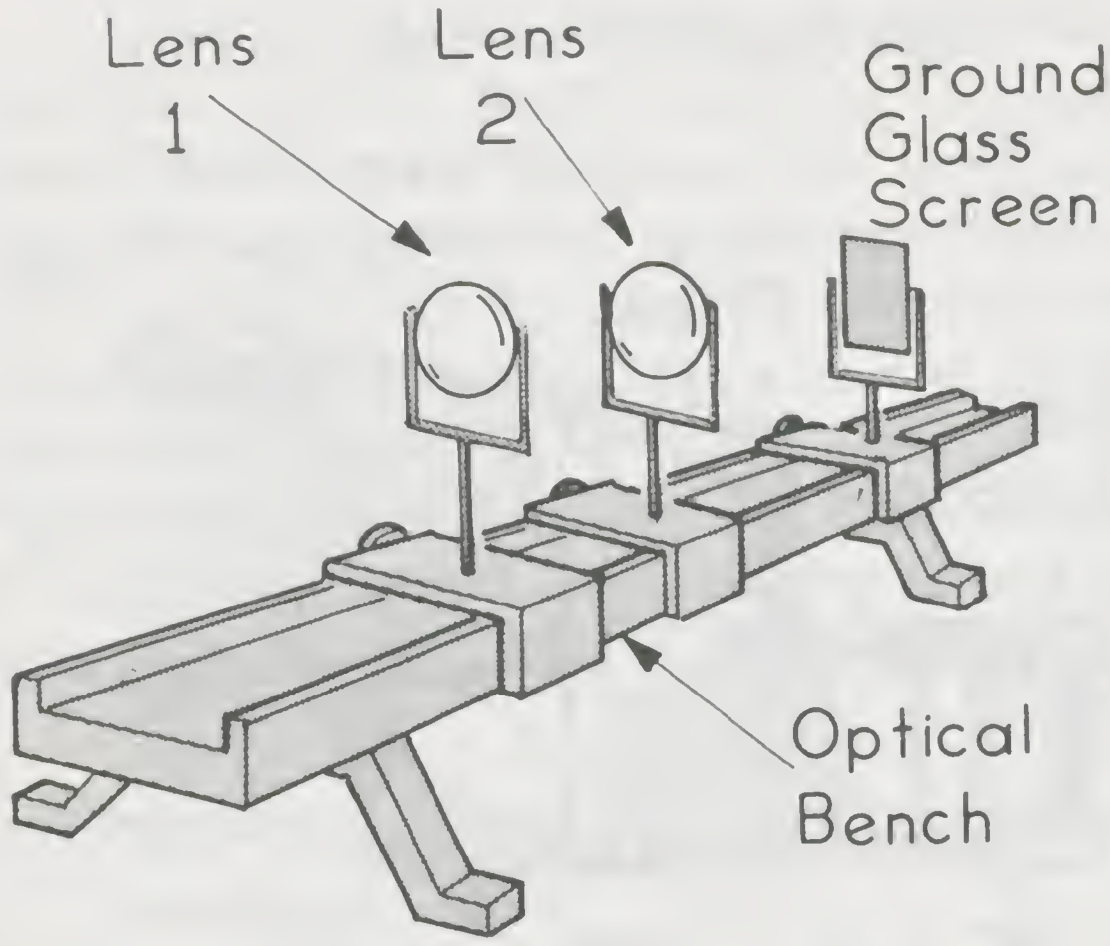

Figure 3.

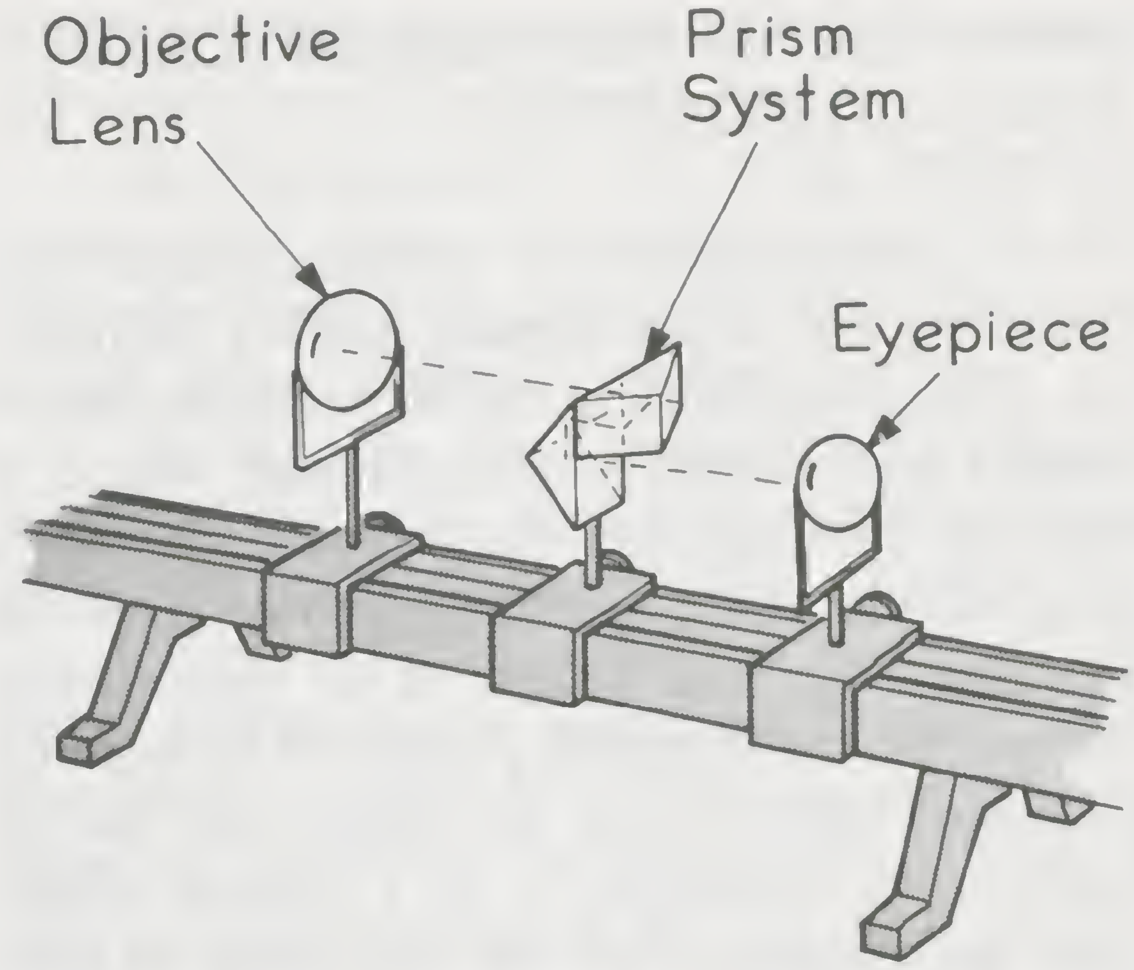

Figure 4.

HOW IS THE MONOCULAR DIFFERENT FROM A TELESCOPE?

In Experiment A-2 you discovered that there is a real image formed inside a monocular. The objective lens, acting exactly like a camera lens, forms a real, inverted image in front of the eyepiece. The eyepiece then acts as a simple *magnifier* (magnifying glass) to examine the real image. The final image formed by the eyepiece cannot be focused on a screen, either inside or outside the instrument; and, therefore, it is a virtual image. You also learned that for the image to look larger through the monocular than it does with the unaided eye, the focal length of the objective has to be longer than that of the eyepiece.

Without the prism assembly, the objective and eyepiece together would form an *astronomical telescope.* A magnified and inverted image is seen when looking through this instrument, since it usually doesn't matter in astronomy if the images of heavenly bodies are upside down. On the other hand, a *terrestrial telescope,* for viewing objects here on earth, usually has another lens inside to invert the image again, making it right side up. The binoculars and monocular differ from telescopes in that they have an internal prism assembly instead of an extra lens, to turn the image over. The prism system also shortens the length of the instrument. There is a series of four reflections in the prism system.

Question 3. Does a simple magnifier give an inverted image?

The Parts of a Monocular

Figure 5 shows a typical binoculars, with a cutaway view of one side of the *body,* so that the internal components can be examined. Note the names of the various parts.

HOW BINOCULARS WORK

When binoculars are used to view an object, light from the object is collected by the objective lens. The objective lens forms a

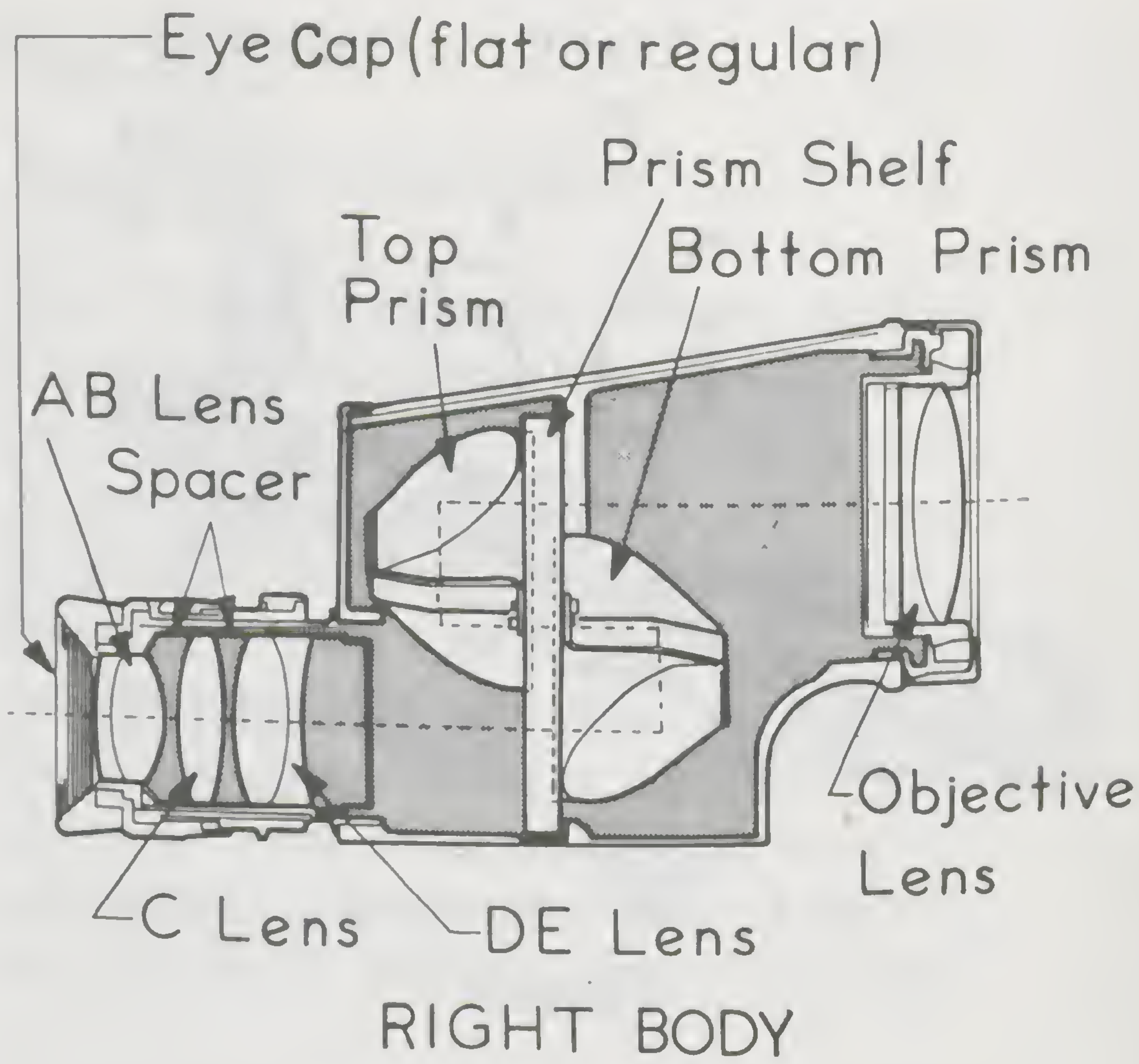

Figure 5. Right body of a binoculars.

real image of the object, but before this image is focused, the light rays which form the image are reflected by two prisms placed in such a way that the image is made upright and moved to a place just in front of the system of lenses forming the ocular (marked as lenses *AB, C,* or *DE* in Figure 5). The ocular then acts as a simple magnifying glass, making the upright image formed by the objective lens and prisms appear larger. Thus the object, when seen through the binoculars, appears larger than it appears to the unaided eye.

LIGHT FROM THE OBJECT

We use binoculars to make distant objects appear larger. The word "object" is used to mean the source of the light which passes through an optical system and forms an image. How does light come from an object to a monocular? Most objects we look at are things which we see by reflected light. That is, they do not produce light in themselves, like incandescent lamps, fires, stars, or the sun. Instead, light strikes the object and bounces off. Light from a given point on some object is usually reflected in many different directions, and only a small part of that reflected light passes into the objective of a monocular.

Figure 6 shows a bundle of light which has been reflected from a point on an object, and then enters the objective. (For simplicity, the object is represented as an arrow standing on end.) Although light is reflected from the point P in all directions, the objective of the monocular intercepts only that part of the light contained within the cone formed from P to the objective. The same thing is true for light coming from each point on the object.

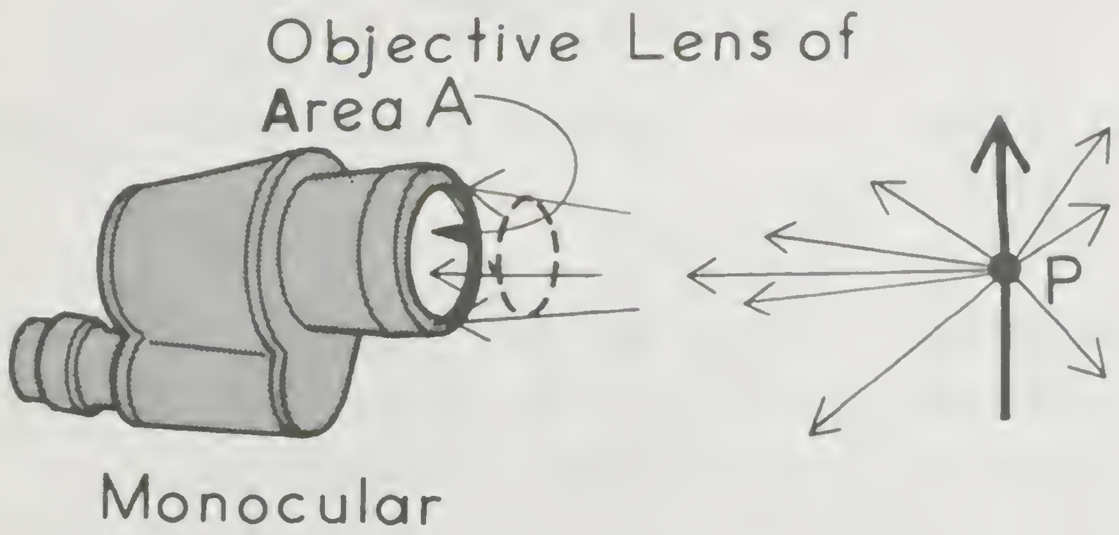

Figure 6.

In *The Camera* module it is shown that the amount of light entering a camera lens is proportional to the square of the diameter of the lens. The same relationship applies to the objective lens of a telescope or monocular.

The light which enters the objective passes through the eyepiece to form an exit pupil, as you observed in Experiment A-1. Thus, the amount of light in the exit pupil is proportional to the square of the diameter of the aperture of the objective lens.

Question 4. In order to double the light gathered by a monocular, how much must the diameter of the objective lens be increased?

Problem 1. How much more light from an object enters the objective lens of a 7 X 50 monocular than enters the objective lens of a 7 X 35 monocular?

Our discussion of light traveling from only one point on an object to the objective of a monocular is helpful, but the actual reflection of light from a small point on some object may not be in all directions. Nevertheless, the average for many points and for *objects at far distances results very nearly in reflection in all directions.*

In this discussion, our basic assumption is that *light travels in straight lines*. The fact that sharp shadows are formed by light from small, distant sources is evidence of this.

We used an arrow as an object in Figure 6. In talking only about the light reflected from one point P on that object, it seems that we ignore the light reflected from all the other points to the objective. Imagine some enormous number N of such points, running from the tip to the feather of the arrow. Each of these points produces its own cone of light, and the total light entering the objective is N times more than from just one point. However, the total light reflected from the object in all directions is also N times greater. The fraction of light entering the objective, compared to the total reflected from all N points, is N times the light intercepted by the objective from one point divided by N times the total light from one point. The N factors then cancel and, therefore, the fraction F is the same for light from all points to the objective as it is from just one point to the objective. This result makes it possible to calculate the fraction of light entering the objective, compared to the total light reflected from an object.

Question 5. Explain why light from a long fluorescent lamp casts a sharp shadow of a pencil when the pencil is held along a direction aligned with the length of the lamp, but casts a fuzzy shadow when the pencil is held at right angles to that direction.

HOW THE LIGHT ENTERS YOUR EYE

You have examined a monocular and followed the path of light from an object through an objective lens, through a prism system and through the eyepiece to form an exit pupil. The light then enters your eye.

In order to get into your eye, the light from a monocular must pass through your eye pupil. As you may know, the pupil of a

human eye gets larger and smaller according to the amount of *ambient* light (the light surrounding it). When you walk out of a dark movie theater into bright sunlight, you experience the sensation of dark-adapted eyes having to change suddenly. Similarly, when you first enter a dark room after having been outside, the room appears to be so dark that you can see nothing. If you wait a few minutes, you can see even in relatively dark areas. Adaptation involves both the change in pupil diameter and the response changes of the retina.

Studies by the U.S. Government during World War II determined that the entrance pupil of the eye is never less than 2.5 mm, even in bright sunlight. The pupil enlarges to about 3.5 mm on a dark cloudy day and further enlarges to about 5.0 mm at dusk, 6.5 mm in full moonlight, and 7 mm in complete darkness.

If the pupil of your eye has a diameter of 4.0 mm, but the exit pupil of the binoculars is 5.0 mm, much of the light collected by the objective is being lost. You will not see the image as brightly as you would if all the collected light could enter your eye. It is, therefore, to no advantage to use binoculars which collect a large amount of light, if much of the light never gets into your eyes.

THE STANDARD SPECIFICATION

In describing binoculars, manufacturers use a *standard specification.* As you have learned, the first number in this specification is the magnification; the second number is the diameter of the objective in millimeters. For example, a 6 X 40 binoculars has a magnification of 6 and an objective lens diameter of 40 mm. For reasons which you will learn later in this module, the diameter of the exit pupil of the monocular eyepiece can be found by dividing the objective diameter by the magnification. For example, the exit pupil from the eyepiece of a 7 X 35 monocular would be 35 ÷ 7 = 5 mm.

Example Problem. What is the magnification and objective diameter of a 10 X 50 binoculars? What is the size of the exit pupil?

Solution. Given is a 10 X 50 binocular designation. The first number is the magnification:

$$M = 10$$

and the second number is the objective lens diameter in millimeters:

$$D = 50 \text{ mm}$$

The diameter of the exit pupil is then 50 mm ÷ 10 = 5 mm.

Problem 2. What is the magnification and objective diameter of an 8 X 35 binoculars? What is the diameter of the exit pupil?

Example Problem. What is the diameter of the exit pupil of binoculars with a standard specification of 6 X 35? What is the cross-sectional area of the exit pupil? If it is dusk when you use these binoculars, and your eye pupil has a diameter of 5.0 mm, what is the cross-sectional area of your eye pupil, and what proportion of the light collected by the monocular enters your eye?

Solution. Given are

$$M = 6,\ D = 35, \text{ and}$$
$$\text{the eye pupil diameter} = 5.0 \text{ mm}$$

The diameter of the exit pupil is then

$$\frac{35 \text{ mm}}{6} = 5.8 \text{ mm}$$

The cross-sectional area of a circle is computed using the formula

$$A = \pi(d/2)^2$$

Substituting the given value of the exit pupil diameter into this equation,

$$A = 3.14 \times \frac{(5.8 \text{ mm})^2}{4}$$

Performing the indicated operations gives

$$A = 26.4 \text{ mm}^2$$

The cross-sectional area of the eye pupil is found in a similar manner:

$$A = 3.14 \times \frac{(5.0 \text{ mm})^2}{4}$$

or

$$A = 19.6 \text{ mm}^2$$

The proportion of the total light through the binoculars which enters the eye is then:

$$\frac{19.6 \text{ mm}^2}{26.4 \text{ mm}^2} = 0.74$$

This result may be written as 74%.

Problem 3. What is the diameter of the exit pupil of binoculars with a standard specification of 8 X 50? What is the cross-sectional area of the exit pupil? If it is dusk when you use these binoculars, and your eye pupil has a diameter of 5.0 mm, what is the cross-sectional area of your eye pupil and what proportion of the light collected by the monocular enters your eye?

Problem 4. You are using 7 X 35 binoculars on a bright sunny day when the pupil of your eye has a diameter of 2.5 mm. What proportion of the light collected by the monocular enters your eye?

SUMMARY

A monocular, one half of the binoculars, makes distant objects appear larger. One of the specifications on the instrument tells how much larger. Typically, two numbers, such as 8 X 50, appear on the instrument housing. These numbers tell, respectively, the magnifying power and the diameter of the objective lens in millimeters. A monocular marked 8 X 50 makes distant objects appear eight times larger and has an objective that is 50 mm in diameter.

The objective lens limits the amount of light entering the monocular; in this capacity it is known as the entrance pupil. The image of the entrance pupil which is formed by the eyepiece is called the exit pupil. The exit pupil is just outside the instrument, very close to the eyepiece, and is at the position where the entrance pupil of any other optical instrument used with the monocular should be placed (e.g., the pupil of your eye). The diameter of the exit pupil can be found by dividing the objective diameter by the magnification.

The objective lens also forms a real image of a distant scene at its focal point inside the monocular. The eyepiece acts as a simple magnifier to examine this real image, which is also at *its* focal point. The final image seen by an observer is a virtual image formed by the eyepiece, of the real image, which is formed by the objective. To make objects look larger than they would with the unaided eye, the objective must have a longer focal length than the eyepiece.

The binoculars or monocular differ from telescopes only because they have an internal assembly of prisms which turn the final image upright and shorten the length of the body by a series of four reflections. There are usually two 45°-45°-90° prisms which, when used in this manner, are called *porro prisms.*

What you have learned about monoculars and about lenses or prisms in this section of the module are *descriptions* of what happens, but not why the instrument works as it does. In the next sections of the module, you will gain a deeper understanding of the binoculars in terms of certain laws of optics.

GOALS FOR SECTION B

The following goals state what you should be able to do after you have completed this section of the module. The example which follows each goal is a test item which fits the goal. When you can correctly respond to any item like the one given, you will know that you have met that goal. Answers appear immediately following these goals.

1. *Goal:* Understand how to apply the laws of reflection and refraction.

   Item: A narrow beam of light shining on a pond forms an angle of between 5° and 30° with the normal to the surface of the pond.

   a. What happens to the light beam at the surface of the pond?
   b. Explain how you would apply the principles or laws which predict this behavior.

2. *Goal:* Be able to solve problems using the law of refraction.

   Item: A plastic block (n = 1.382) is tightly bonded to a glass block (n = 1.520). A ray of light is incident at the boundary of the two materials, going from plastic into glass. If the incident ray makes an angle of 30° with the normal, what is the angle between the refracted ray in the glass block and the normal?

3. *Goal:* Know how to find the principal planes of a compound converging lens.

   Item: Locate a camera lens system, or converging lens system from some other optical device or instrument. Using the lens system, find the positions of the principal points and describe the principal planes.

4. *Goal:* Be able to construct a principal ray diagram for a converging compound lens.

   Item:

   a. For the lens system of Item 3, construct a principal ray diagram, and use the diagram to predict the location and height of the image of a 25-watt light bulb. (The bulb is located at a distance of twice the focal length away from one of the focal points.)
   b. Check your prediction by having a fellow student or your teacher find the image on a screen. The actual position should be within ±2 cm of your prediction.

5. *Goal:* Know how to construct a principal ray diagram for a diverging lens.

   Item: A certain diverging lens has principal planes separated by 2 cm. The lens has a focal length of 6 cm. A pencil 6 cm long is placed in a vertical position with the eraser on the principal axis. The eraser is 18 cm from the object focal point and on the opposite side of the lens.

   a. Construct a principal ray diagram.
   b. Determine the location and height of the image of the pencil.
   c. How far is the pencil from its image?

6. *Goal:* Understand the phenomenon of dispersion.

   Item: A ray of white light is incident on the boundary of glass and air, from the inside of the glass. The angle of incidence is precisely the critical angle for green light. Describe what happens to the red part and the blue part of the incident light ray.

Answers to the Items Accompanying the Preceding Goals

1. a. At the surface, some of the light beam is reflected, and some is refracted. Both beams lie in the same plane as the normal and the incident ray. The angle between the normal and the reflected ray is equal to the angle between the normal and the incident ray. The refracted beam is bent toward the normal.

   b. To calculate the angle of refraction, the sine of the incident angle would be divided by the index of refraction of water. This result is the sine of the refracted angle. Using the sine table, the angle of refraction can then be determined.

2. $\theta_r = 27°$.

3. Mount the lens system on an optical bench (or on a meter stick). Using a distant object, locate the positions of the focal points on both sides of the lens. Next place an object a short distance beyond one of the focal points and locate its image. Measure the image and object distances, x and x'. The focal length is then, $f = \sqrt{xx'}$. Using this value of focal length measure a distance, f, from each focal point back toward the lens. These positions are the principal points. The principal planes contain these points and are normal to the lens axis.

4. a. The image will be at a distance of one half the focal length away from the image focal point. The image will be one half as high as the object.

5. b. The image distance is 2 cm. The image height is 2 cm.

   c. The image is located 10 cm from the object nearer the lens.

6. The green part of the light ray has an angle of refraction of 90°. Because the index of refraction for red light is less than that for green light, the angle of incidence is not great enough to be at the critical angle for red light. Thus, the red part of the light passes out of the glass block. Because the index of refraction for blue light is greater than that for green light, the critical angle for blue light is smaller than that for green light. Thus the blue part of the light is totally internally reflected.

SECTION B

Physics Principles of Binocular Components

INTRODUCTION

In this section of the module, you will learn physics principles which explain how lenses bend light and how light is reflected from porro prisms. You will also learn why compound lens systems are necessary and how to locate the images produced by such systems. These same methods and principles apply to almost all optical devices.

Reflection and Refraction

The bending of light by a lens is a result of a more general phenomenon called *refraction.* If you have completed *The Camera* module, you have learned how refraction occurs, but you have not yet investigated the law of refraction in a quantitative way. In the porro prism system, light is reflected in a process which is related to refraction.

You are now going to investigate two important relationships: the relationship between a reflected light ray and the incident ray, and the relationship between a refracted light ray and the incident ray.

EXPERIMENT B-1. Reflection and Refraction

In this experiment we wish to examine what happens to a light ray when it is reflected and when it passes from one medium to another. You have been provided a Hartl optical disc (or another ray apparatus), as shown in Figure 7.

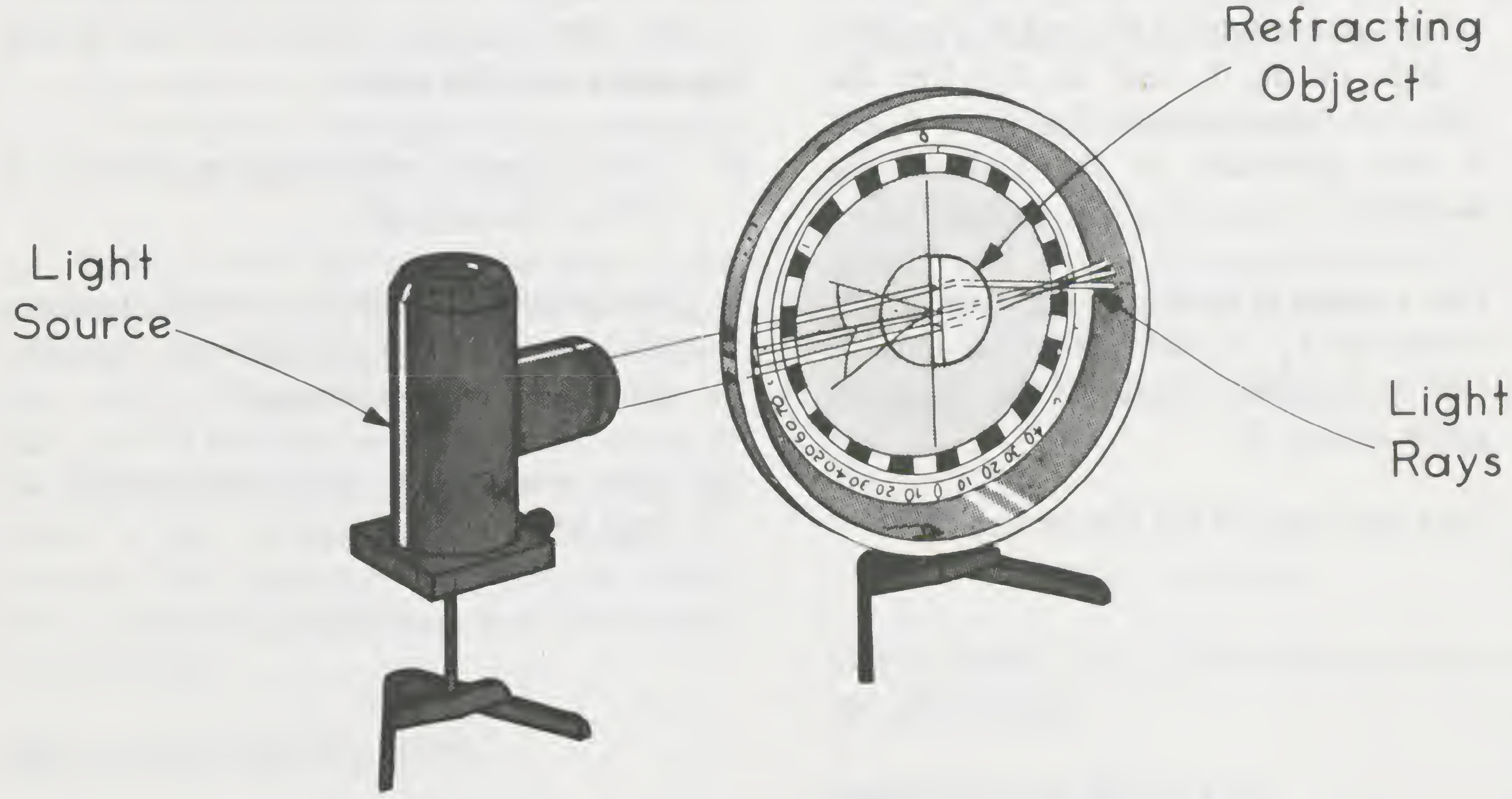

Figure 7.

Arrange the optical disc and light source so that you have a single ray of light passing through the center of the disc. Place the semicircular glass plate so that the center of the flat edge is at the center of the disc. Turn the disc so that the incoming ray of light strikes the flat surface of the glass plate at an angle of 90°.

The object is to measure sets of three angles. All three are to be measured from the line which is *normal* (perpendicular) to the surface of the flat edge of the glass plate as shown in Figure 8.

If you turn the disc slightly, you will see that part of the incident ray is reflected at the glass while part of it passes through the glass. You will measure the angle for the incident ray (θ_1), the reflected ray (θ_r), and the ray passing through the glass (θ_2). The ray which passes through the glass will not bend as it leaves the glass, since this ray is always normal to the curved edge. Start at 0° for the incident ray and measure the three angles for 5° increments of θ_1. Record this information in Table I in the work sheet.

You should now take out the work sheets at the back of this module. Write answers to questions and complete the tables on those sheets as you do the experiment.

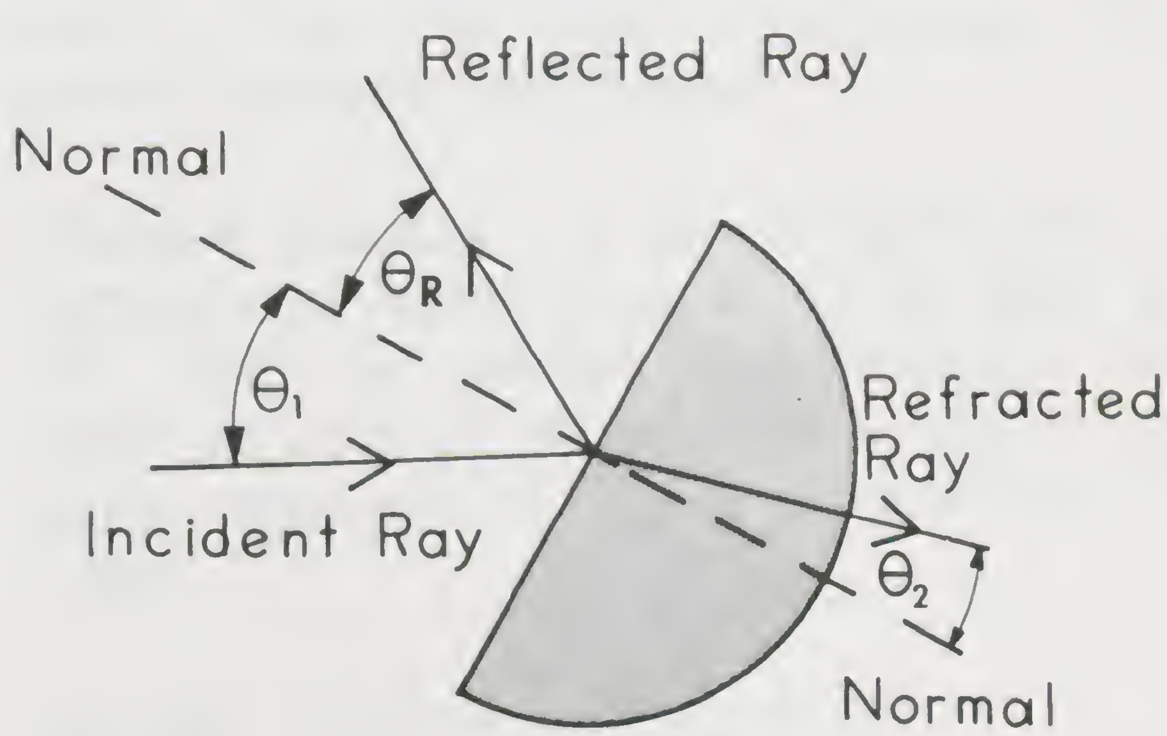

Figure 8.

1. What is the relationship between θ_1 and θ_r?

2. Can you determine a relationship between θ_2 and θ_1?

3. Plot a graph of θ_1 on the vertical axis versus θ_2 on the horizontal axis. Is this graph a straight line?

4. Using your values of θ_1 and θ_2, prepare a table of sin θ_1 and sin θ_2. Use the table of trigonometric functions found in the appendix at the end of the module.

5. Plot a graph of sin θ_1 on the vertical axis versus sin θ_2 on the horizontal axis. Is this more nearly a straight line than the graph of step 3?

6. Find the slope of the line in step 5.

7. Write a relationship between sin θ_1 and sin θ_2.

Turn the optical disc so that the incoming ray strikes the curved edge of the glass plate. Observe both the ray that is relfected internally at the flat edge (this ray passes back through the glass) and the ray that passes out of the glass at the flat edge. Rotate the disc so that the angle of the ray which is incident on the flat edge increases. Observe the ray leaving the glass at the flat edge.

8. What happens when this ray emerges at 90° to the normal?

9. Observe the reflected ray as you turn the disc past that point. Does the intensity of the reflected ray change?

10. Can you explain your observations of step 9?

THE LAW OF REFLECTION

From Experiment B-1, and many similar experiments which have been done by others, we can conclude that light follows two laws of reflection from smooth surfaces:

1. The reflected ray lies in the plane of the incident ray and the normal to the reflecting surface.

2. The angle of incidence equals the angle of reflection, the angles being measured from the normal.

Refraction is the name we give to the bending of a ray of light at the boundary between two transparent materials. When light strikes a smooth surface like water or glass, part of the light is reflected according to the laws of reflection, and part is refracted (bent) at the surface as it passes into the medium. The angles of incidence and refraction are measured with respect to the normal to the surface.

THE LAW OF REFRACTION

As you found in Experiment A-2 for refraction, the ratio $\sin\theta_1/\sin\theta_2$ is constant.

Thus:

$$\frac{\sin\theta_1}{\sin\theta_2} = \text{constant} \qquad (1)$$

This is the *law of refraction* and is usually called *Snell's Law.* The constant in Snell's Law is given a name, the *relative index of refraction* between the two media. If light passes from a vacuum into some material, we call the constant the *absolute index of refraction* of the material. This quantity, commonly called just the *index of refraction,* is a property of the material itself. We define the index of refraction of a vacuum as exactly one. Actually, there is very little bending of light when it passes from a vacuum into air, so that for most measurements of the index of refraction of a material, air may be used as one of the media, instead of a vacuum. We use the symbol n to represent an index of refraction.

If we consider light traveling from a vacuum ($n = 1$) to medium 1, we have

$$\sin\theta_V/\sin\theta_1 = n_1 \qquad (2)$$

and for light going at the same incident angle θ_V from a vacuum into medium 2,

$$\sin\theta_V/\sin\theta_2 = n_2 \qquad (3)$$

Now, dividing the left sides of Equations (3) and (2) and setting that quotient equal to the quotient of the right sides, we get

$$\sin\theta_1/\sin\theta_2 = n_2/n_1 \qquad (4a)$$

or

$$n_1\sin\theta_1 = n_2\sin\theta_2 \qquad (4b)$$

This is Snell's Law of Refraction between two specified media.

Applications of Snell's Law

Snell's Law relates the angle of incidence, θ_1 , and the angle of refraction, θ_2 , with the indices (plural of 'index") of refraction n_1 and n_2 of two media. To use this law, refer to the table of trigonometric functions in the appendix.

Question 6. If θ_2 were the angle of incidence and θ_1 were the angle of refraction, how would Equation (4b) change?

Example Problem. A ray of light enters a container of water from air at an angle of 42° from the normal. The refracted ray makes an angle of 30° with the normal in the water. Determine the index of refraction of water.

Solution. Given in the problem are

$$\theta_a = 42° \text{ and } \theta_w = 30°$$

Equation (2) may be used to determine the index of refraction of medium 1, in this case water. The sines of the two angles must first be determined, and from the tables of trigonometric functions. They are $\sin 30^\circ = 0.50$ and $\sin 42^\circ = 0.67$. Substituting these values into Equation (2) gives

$$n_1 = \frac{0.67}{0.50}$$

Performing the indicated division gives

$$n_1 = 1.33$$

for the index of refraction for water.

Problem 5. A ray of light enters a piece of flint glass from a vacuum at an angle of 46° from the normal. If the refracted ray in the glass is at an angle of 26° with the normal, what is the index of refraction for flint glass?

Problem 6. A light ray traveling in alcohol strikes the surface of a piece of flint glass at an angle of 37° from the normal. If the indices of refraction for alcohol and flint glass are 1.36 and 1.63, respectively, at what angle with the normal to the surface does the light ray travel in the flint glass?

Question 7. A golfer drives into the rough, where the ball comes to rest in a puddle of water. Since he is close to the green, he decides to try a shot from where the ball lies. If the ball is under four inches of water, and the golfer is six feet tall, will refraction cause him difficulty in hitting the ball? If the ball is hit, describe which part of the club will probably strike the ball. Can you figure out what the shot is likely to do in this circumstance?

Refraction and Lenses

How does a ray of light bend when it passes through a lens? You could answer this question by applying Snell's Law at each surface of the lens. But you would have to do this for several rays, before you could locate an image. This would be a difficult and time-consuming process. Doing this one can derive an equation which gives the value of the focal length of a thin, spherical lens in terms of the index of refraction of the glass and the radii of curvature of the two lens surfaces. If you are interested in this derivation, you can find it in textbooks on geometrical optics. The important point is that application of Snell's Law can give the focal length for any lens or lens system.

Thin, Converging Lenses

The following statements summarize the major principles of converging lenses which are developed in *The Camera* module. They are illustrated in Figure 9.

Rays of light which are parallel to the axis of a thin converging lens are converged on the other side of the point of the lens to a point which is called a *focal point.*

The distance from a thin lens to a focal point is called the *focal length* of a lens.

The distance from the focal point of a converging lens to an object located on the same side of the lens is called the *object distance.* Object distance is represented by the symbol x.

The distance from an image produced by a converging lens to the focal point on the same side of the lens is called the *image distance.* Image distance is represented by the symbol x'.

The relationship of image distance to object distance for a lens of focal length f is given by the empirical equation

$$xx' = f^2$$

The *lateral magnification* of a converging lens is defined as the ratio H_i/H_o, where H_i is the height of the image and H_o is the height of the object. Values of lateral magnification may be less than, equal to, or greater than one; thus an image may be smaller than the object, the same size as the object, or larger than the object.

The relationship of lateral magnification, H_i/H_o, to object distance for a lens of focal

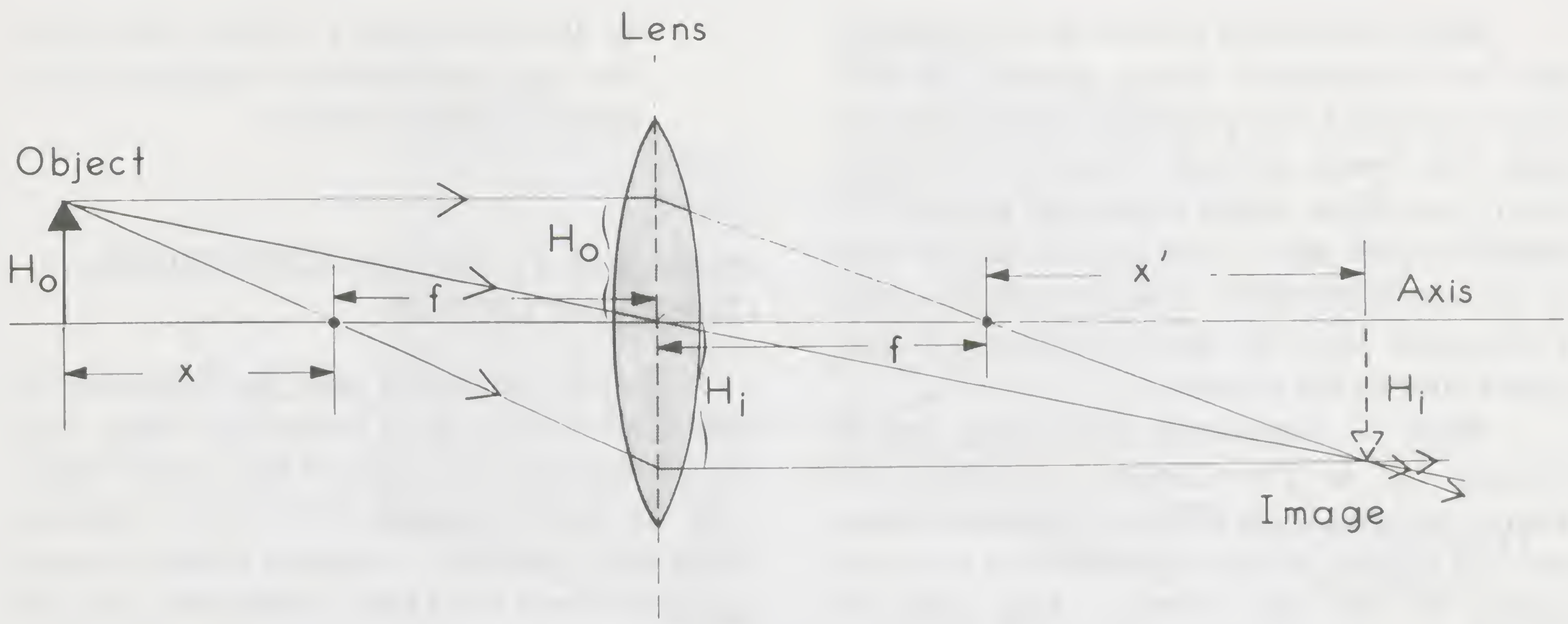

Figure 9.

length f is given by

$$H_i / H_o = f/x$$

The position and height of an image produced by a converging lens can be found by the *method of Principal Ray Diagrams.* This method consists of the following steps, as shown in Figure 9:

1. Draw a vertical line at the position of the lens, a lens axis, and focal points located at convenient distances, scaled to the focal length of the lens.

2. Draw an object with its base on the axis and its height reduced by the scaling factor selected in step 1. Place the object at an object distance scaled by the same factor.

3. Construct a line from the top of the object parallel to the axis to a point on the vertical line representing the lens. From this point, the line should be constructed to pass through the focal point on the other side of the lens from the object.

4. Construct a line from the top of the object through the focal point on that side of the lens to a point on the vertical line representing the lens. From this point, construct the line parallel to the lens axis.

5. Construct a straight line through from the top of the object through the point of intersection of the vertical line and the lens axis.

6. The three principal rays you have drawn should intersect at one point. This point is the image point corresponding to the top of the object. Draw in the rest of the image.

7. Measure the scaled image height and scaled image distance and use the scale factor to find the actual image height and actual image distance.

Compound Lenses

As shown in Figure 5, a real monocular uses compound lenses, *not* thin lenses. The monocular shown has what appear to be three lenses for the eyepiece and one lens for the objective. However, upon closer examination, one can see that the eyepiece lens nearest the eye (the *AB* lens) really consists of *two* lenses placed together. The objective lens also consists of paired lenses.

Since real optical devices like the binoculars use compound lenses instead of thin lenses, we need lens principles which apply to such real lenses instead of just thin lenses. Also, one of the lenses in the pair making the objective (and one of the pair in the *AB* lens of the eyepiece) is *not* a converging lens. It is a *diverging* lens. We shall examine diverging lenses later in the module.

When a compound converging lens is studied (as in *The Camera* module), it is found that the focal points of the lens system are *not* located at the same distance from the center of the lens system. This observed property of compounded lenses means that one cannot measure focal length as the distance from the lens to the focal point. But it *is* possible to locate focal points, and to calculate a focal length by using object and image distances in the equation $xx' = f^2$. As stated in the summary principles of Section C of *The Camera* module:

> For a compound lens the focal length cannot be found simply from measuring the distance from a focal point to a "lens." For a compound lens the focal length is given either by xx' or by the expression $f = H_i/\alpha$, where α is the angle subtended (in radians) at the lens by a distant object.

PRINCIPAL PLANES OF CONVERGING COMPOUND LENSES

Suppose that you find the locations of the focal points of a compound lens, and then determine the value of the focal length, *f*, by the method used in *The Camera* module. When you measure a distance *f* back toward the lens system from each focal point, you are at the positions of what are called the *principal planes.* Figure 10 shows the principal planes for a compound lens having asymmetrical focal points. Here one of the principal planes is inside the compound lens and the other is outside it.

For a thick lens with symmetric focal points the principal planes might be located as shown in Figure 11.

As we select thinner and thinner lenses, the principal planes move closer until they are together at the center of a thin lens, as shown in Figure 12.

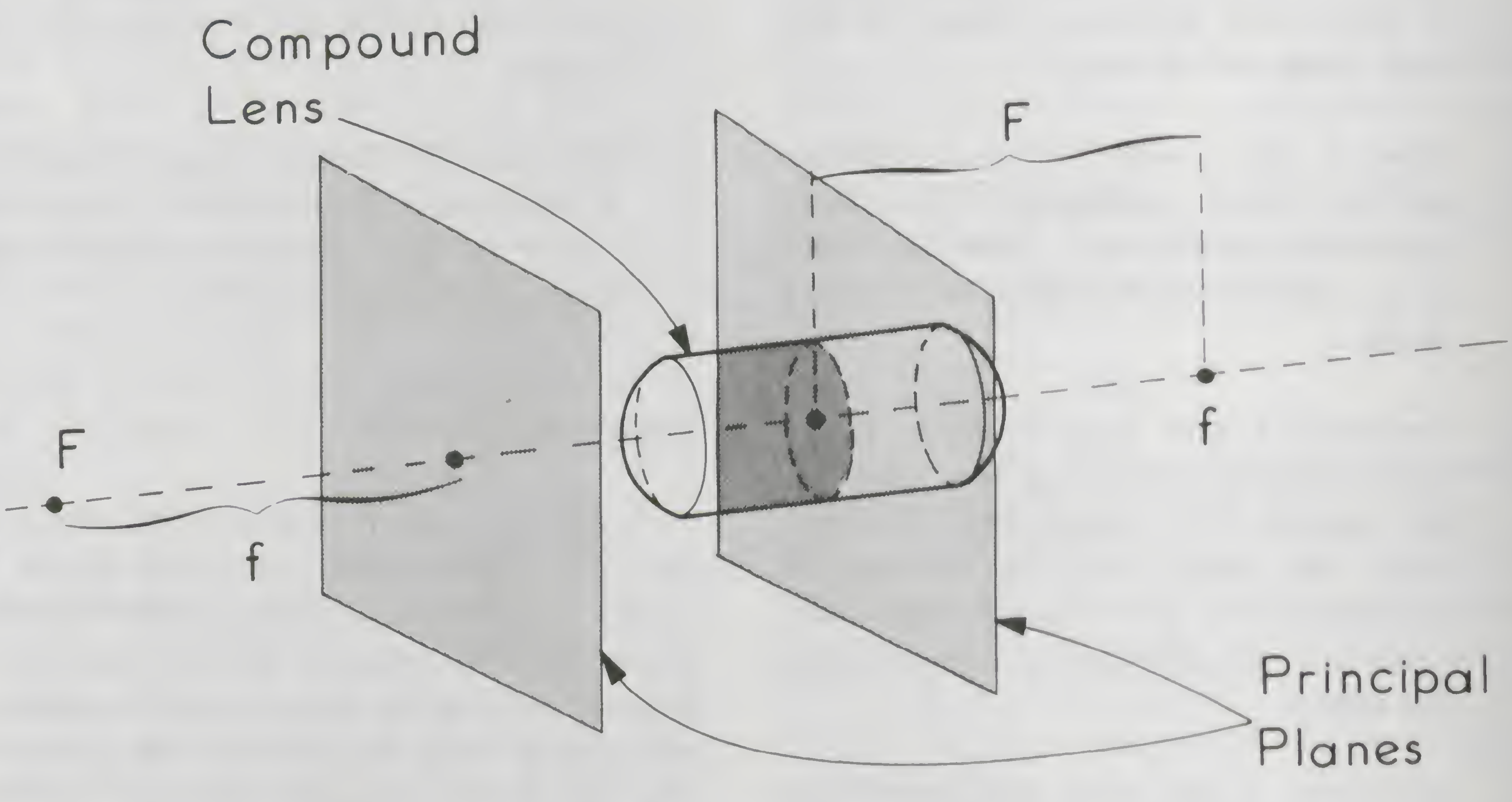

Figure 10.

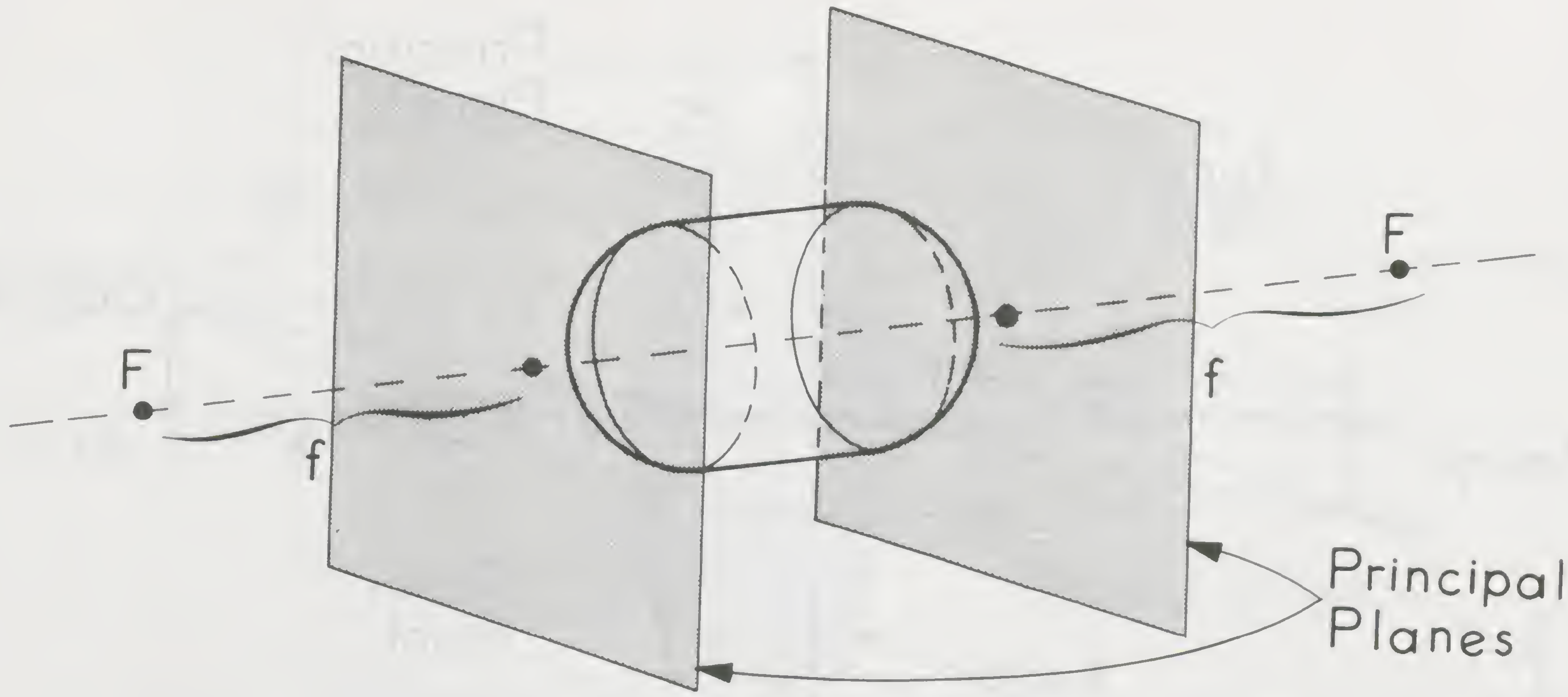

Figure 11.

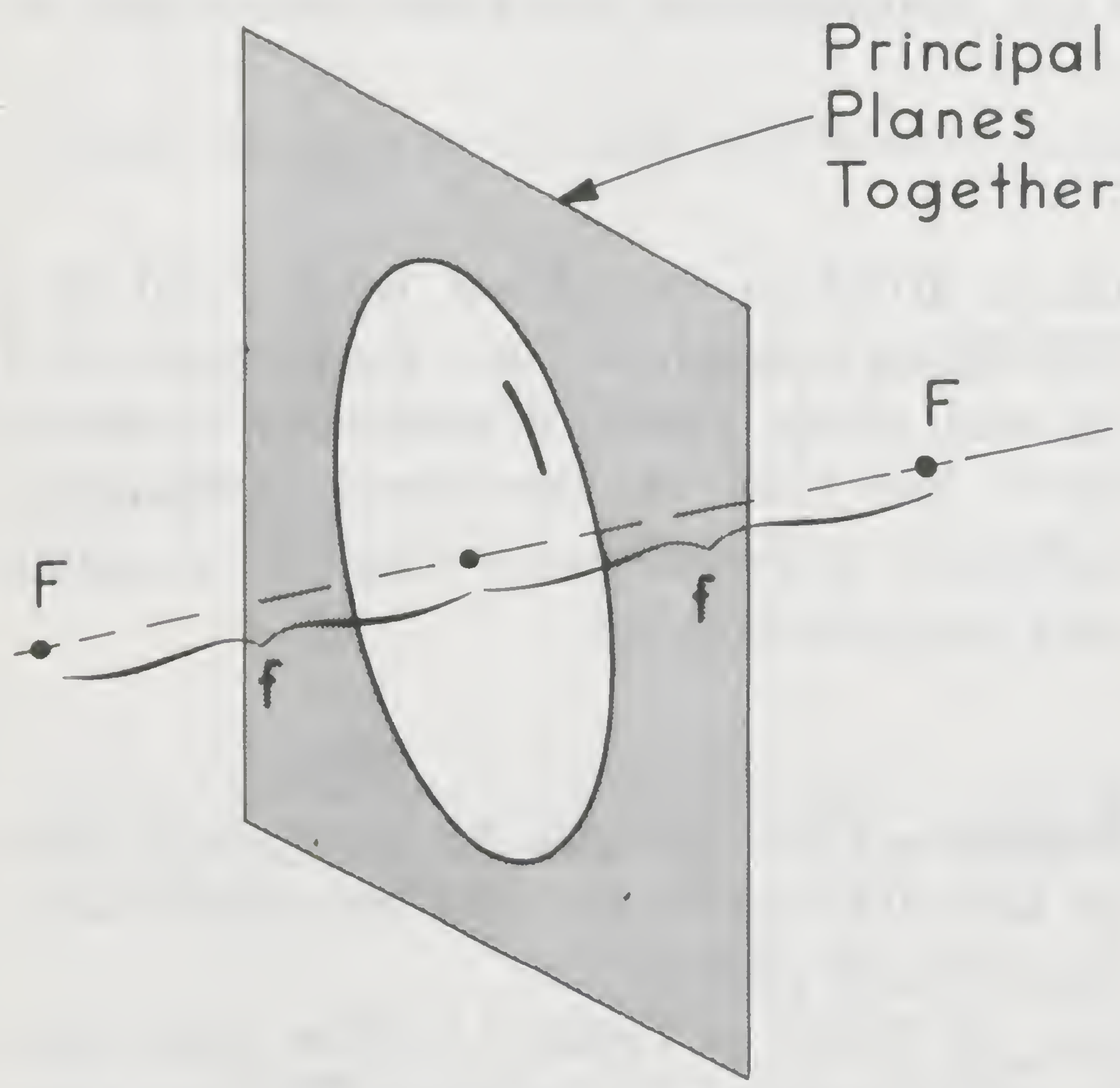

Figure 12.

As you can see, for a thin lens the focal length can be measured from the lens (really from its center) to the focal point.

PRINCIPAL RAY DIAGRAMS FOR CONVERGING COMPOUND LENSES

The equation

$$xx' = f^2$$

can be used to locate an image for a given object position with respect to a converging compound lens. Also, the equation

$$H_i / H_o = f/x$$

can be used to find the lateral magnification of an image produced by such a lens system.

From these equations a method of principal ray diagrams may be arrived at for converging compound lenses. The correct method has only to take into account the use of principal planes.

Figure 13 shows the principal planes and certain principal rays drawn for a converging lens system.

The positions of the two principal planes are found by laying off distances equal to the focal length, from the focal points toward the lens. You would determine the location of the focal points and the value of the focal length by the methods used in *The Camera* module. (That is, find the focal points by focusing distant objects on a screen, then find the focal length by measuring x and x' for nearby objects.) The intersections of the principal planes with the axis are called *principal points.* In the ray diagram, the lens may be replaced by its two principal planes. In the diagram, all bending of the light rays is made to take place at the principal planes, although in reality the rays bend at the surfaces of the lens. A given lens or lens system has principal planes located at very definite places, and the

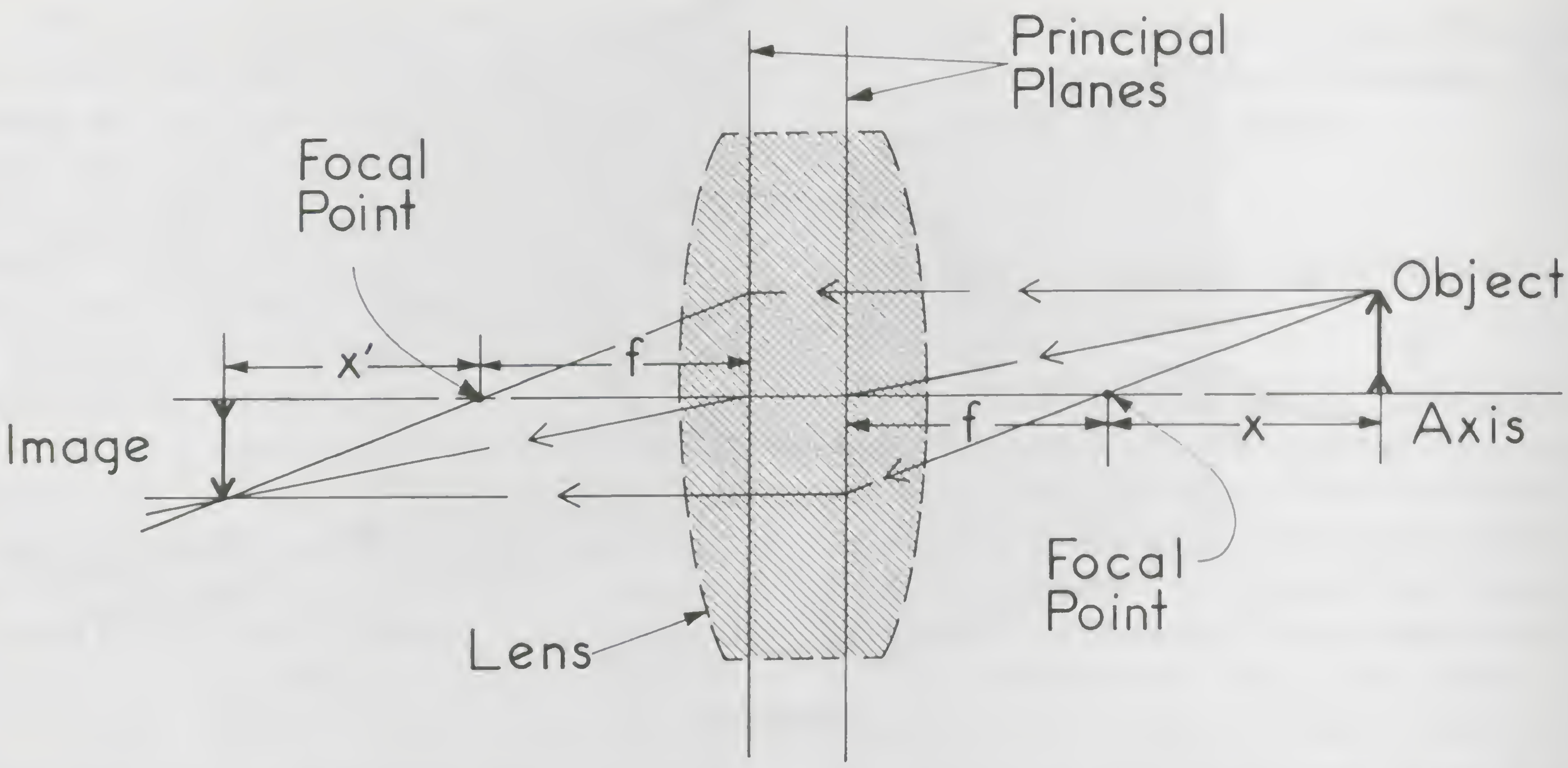

Figure 13. A ray diagram for a converging lens, showing the axis, principal planes, focal points, focal lengths, the object, and image sizes and locations.

position of the image along the lens axis depends on the separation of the principal planes.

Although an infinite number of light rays from any point on the object pass through the lens, only two or three conveniently chosen rays are used for ray tracing. As in Figure 13, three rays from a point from the object can be chosen: one ray parallel to the axis, one ray to the first principal point, and one ray through the focal point. From the first principal plane, all rays are drawn parallel to the axis until they strike the second principal plane. The ray which was originally parallel to the axis then bends so that it passes through the image focal point. The ray which went through the first (and second) principal point emerges parallel to its original direction. Finally, the ray that went through the first focal point emerges parallel to the axis. These three rays intersect at a point on the side of the lens opposite the object; this point of intersection is the point on the real image which corresponds to the point on the object from which the rays came.

Example Problem. Locate the image by means of a principal ray diagram, and find its height for an object of height 1 cm located 5 cm in front of the object focal point of a converging lens which has a focal length of 4 cm and whose principal planes are separated by 1 cm. (Let the lens have a diameter sufficient to permit you to use any desirable rays from the object.)

Solution. The principal ray diagram is done by first drawing an axis and two parallel lines for principal planes.

As shown in Figure 14, these lines must be perpendicular to the axis. They are separated by 1.0 cm. (Figures 14 through 17 are printed full scale. Figures 18 through 24 have been slightly reduced.)

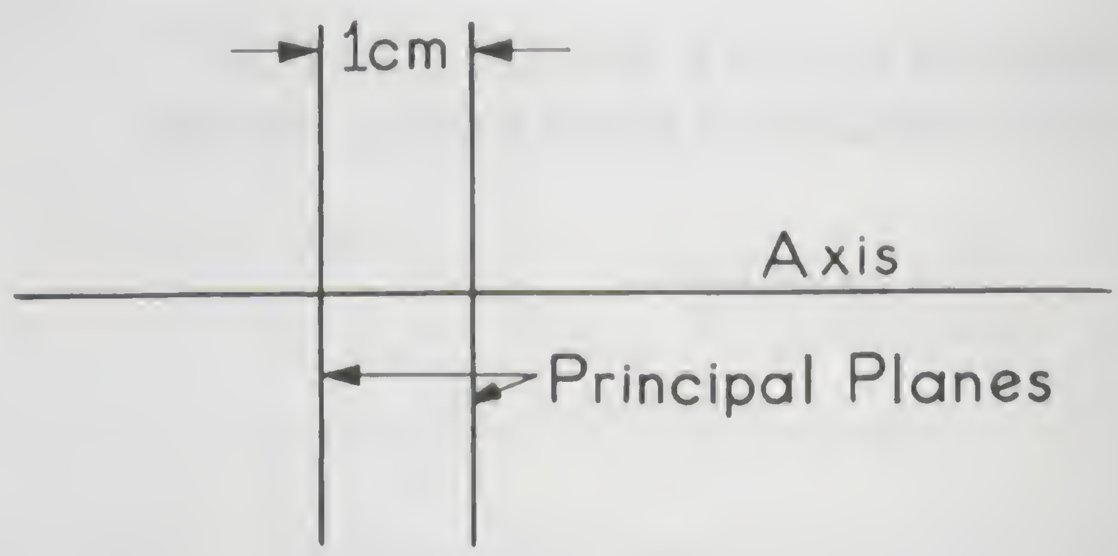

Figure 14.

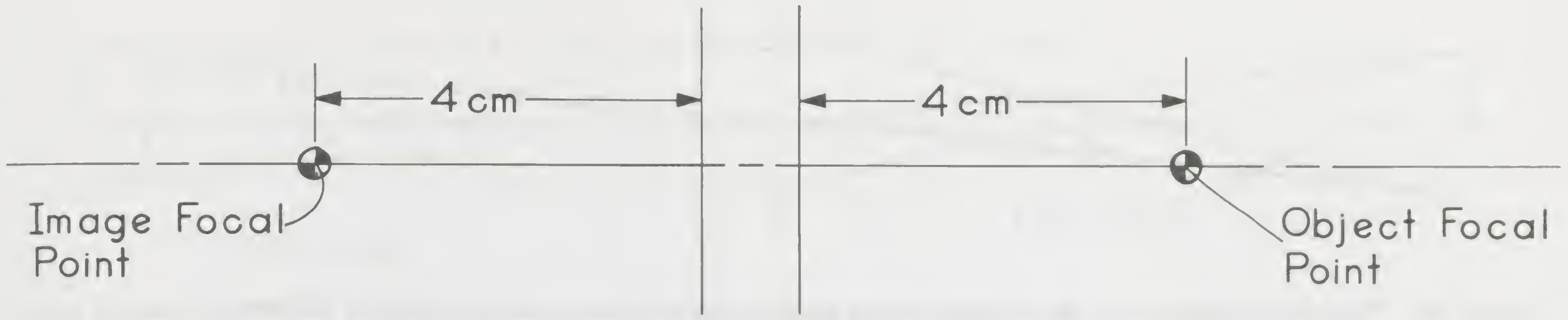

Figure 15. Then the focal points are located by using a rule graduated in centimeters.

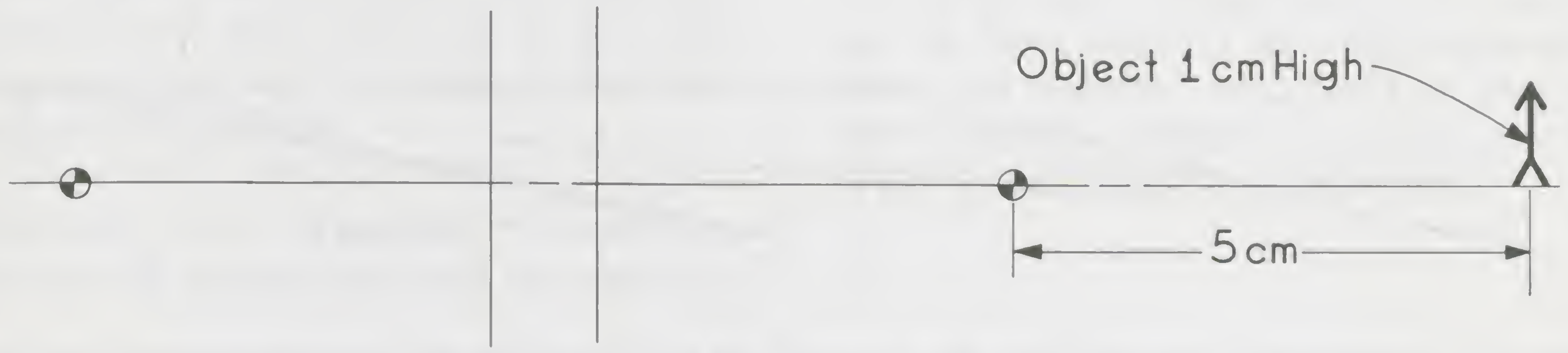

Figure 16. An arrow 1 cm high may be used as an object and it may be placed 5 cm from the object focal point.

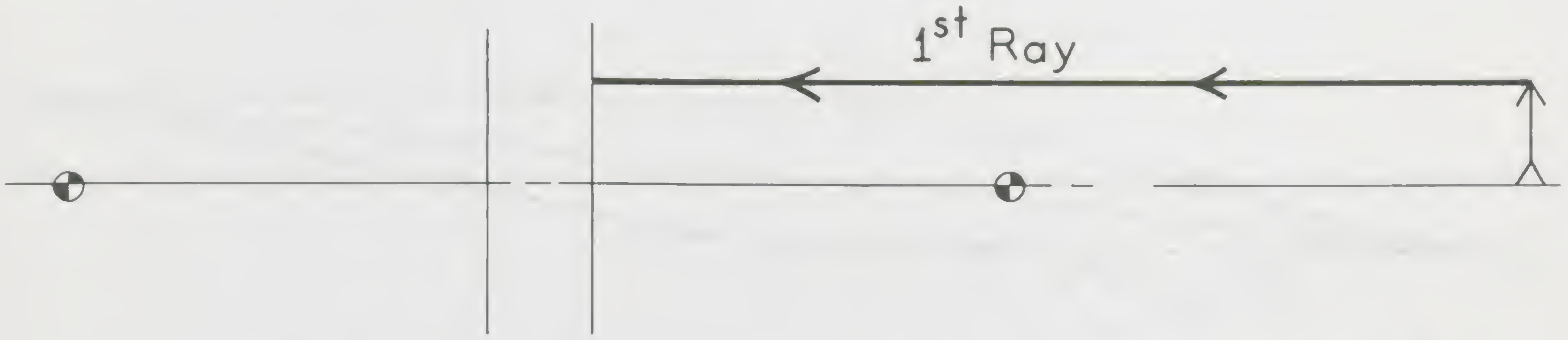

Figure 17. The first ray is drawn parallel to the axis from the top of the object to the first principal plane. All rays must pass parallel to the axis on going from one principal plane to the other.

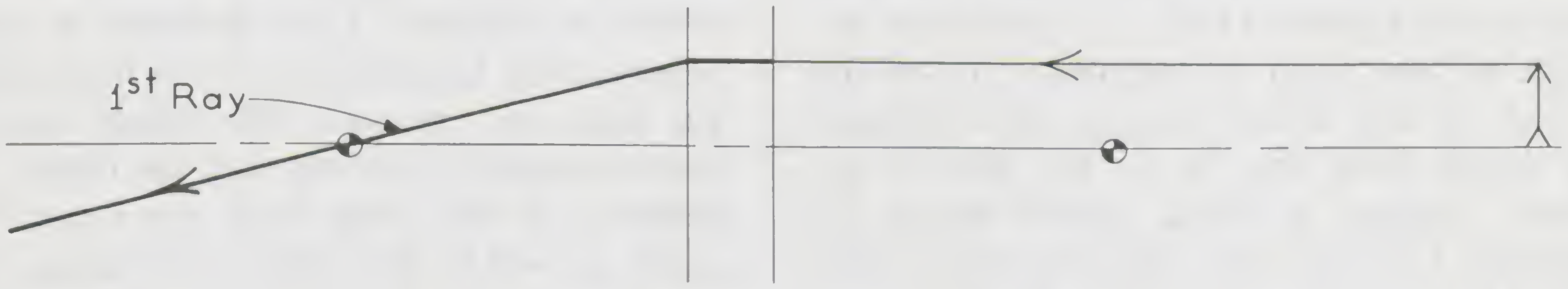

Figure 18. This ray is shown bending at the second principal plane (in reality it bends at all lens surfaces) and passing through the image focal point.

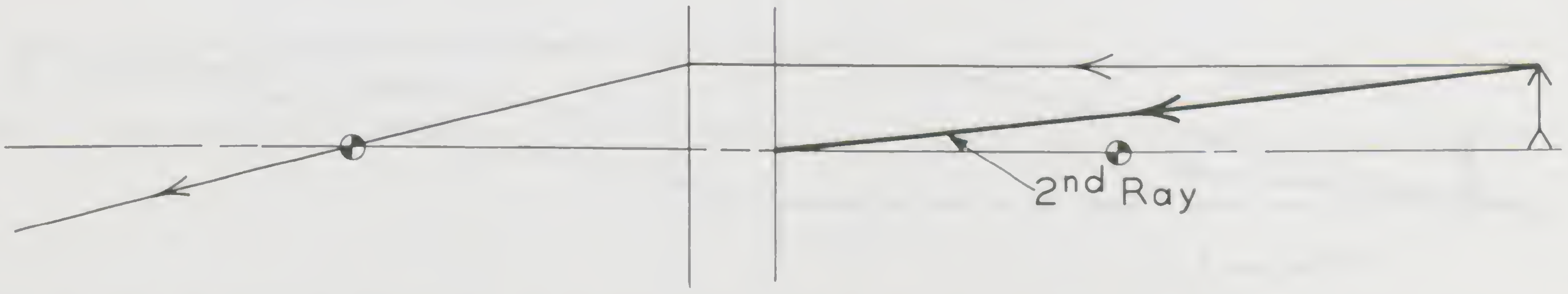

Figure 19. The second ray begins at the same spot on the object and goes to the first principal plane at its intersection with the axis (principal point).

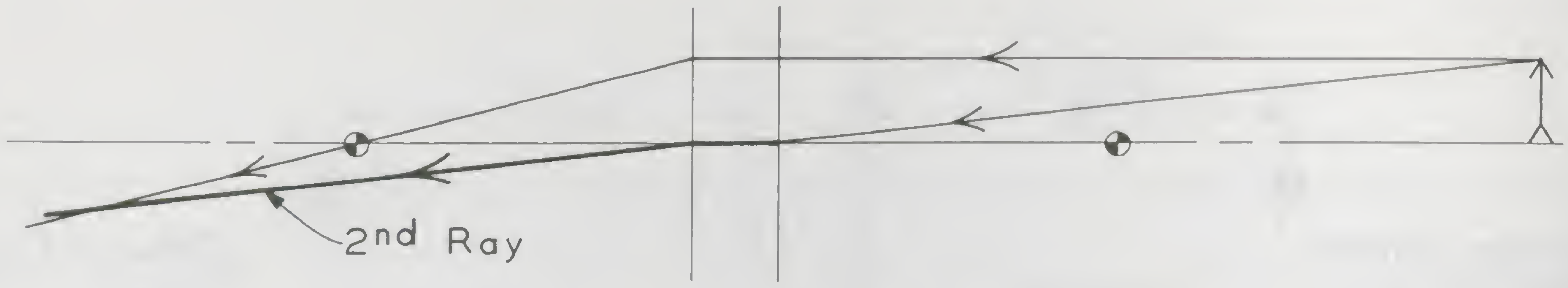

Figure 20. This ray must bend at the first principal plane so that it can travel to the axis between principal planes (in this case *on* the axis). Then it must bend again at the second principal plane so that it leaves the lens parallel to its original direction.

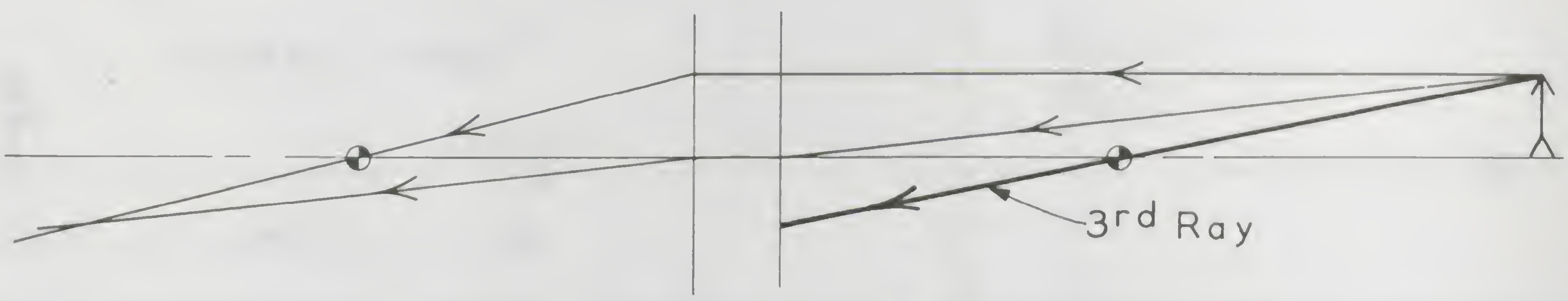

Figure 21. The third ray also starts from the same place on the object. This ray travels through the object focal point to the first principal plane.

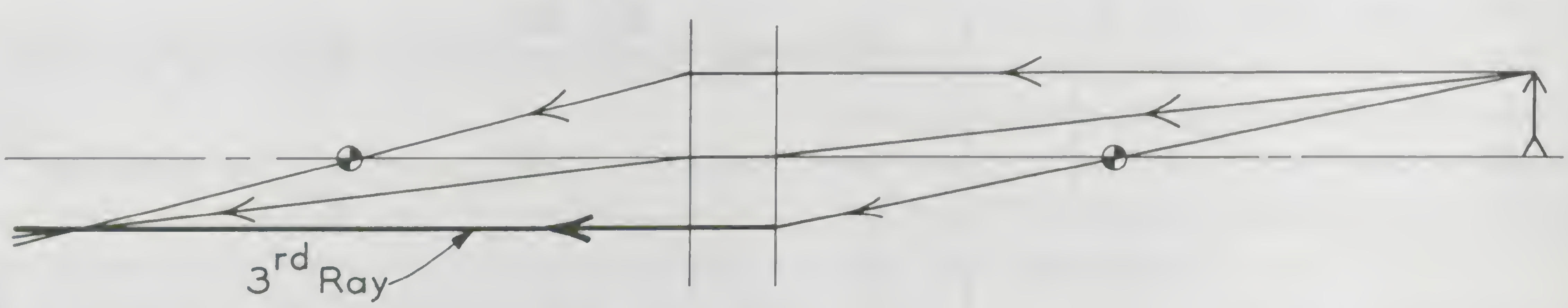

Figure 22. The ray then bends parallel to the axis, passes between the principal planes, and emerges parallel to the axis from the second principal plane.

The intersection of the three rays constructed in Figures 14 to 22 constitutes the image of that point on the object (tip of the arrow). If this entire process were repeated for points from the tip to the base of the object, images of those points would be found in a vertical line. Since the base of the object lies on the axis, the image of this base lies on the axis.

The image may now be constructed, as shown in Figure 23, by drawing an arrow between the intersection of the three principal rays and the axis. The image must be perpendicular to the axis, as is the object. The distance of this image from the image focal point, as well as the height of the image, may now be measured in centimeters. (Remember, there is a scale factor of 75%.)

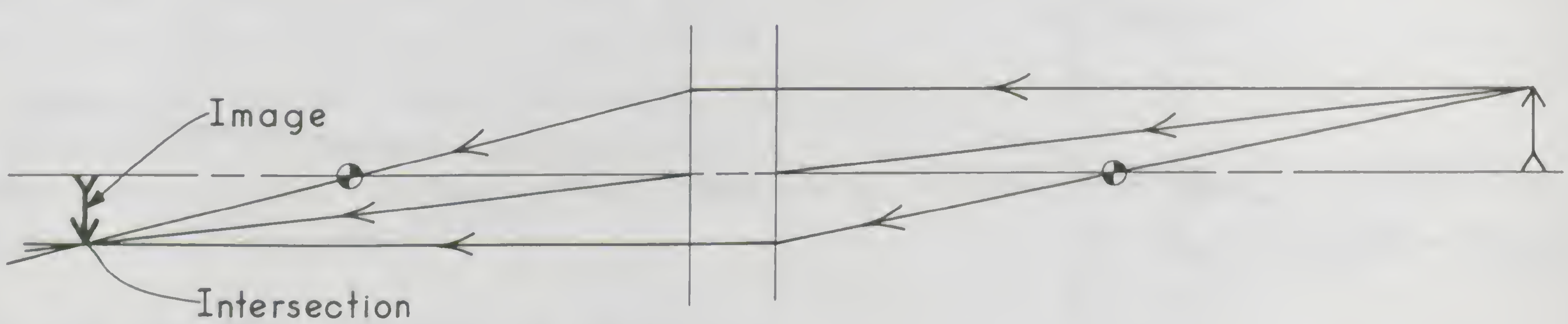

Figure 23.

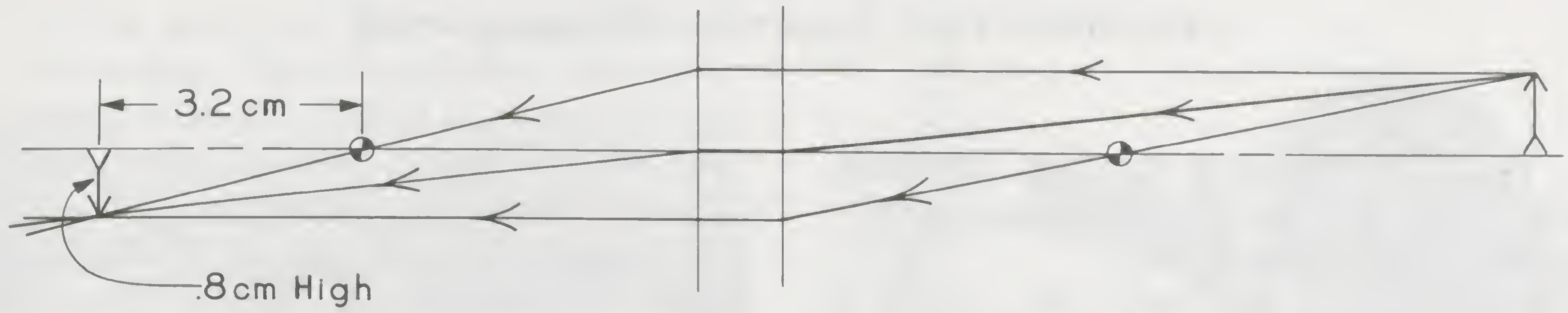

Figure 24. The image distance shown is 3.2 cm and the height of the image is 0.8 cm.

Problem 7. Using the equations, $xx' = f^2$ and $H_i/H_o = f/x$, show that the results just found in the example agree with those found by the use of equations and, therefore, that the two methods are equivalent.

Problem 8. Locate the image by means of a principal ray diagram, and find its height for an object of height 3 cm located 10 cm in front of the object focal point of a converging lens of focal length 5 cm, whose principal planes are separated by 2 cm. (Let the lens have a diameter sufficient to permit your use of any desirable rays from the object.)

EXPERIMENT B-2. Properties of Diverging Lenses

You are now ready to study diverging lenses, which are used in many optical devices, including binoculars.

You have been provided with an optical bench, a light source which serves as an object, and a diverging lens.

Place the light source (object) near one end of the bench. Place the lens in a holder near the middle of the bench. Place a screen on the opposite side of the lens and try to find a clear image. Can you find an image on the screen?

Now remove the screen and look through the lens. Since you can see the light source (object) clearly through the lens, it must be forming some kind of image. However, this kind of image can never be formed on a screen (try it and see), and it is therefore called a *virtual image,* as opposed to a *real image,* which you have seen before. Virtual images can be viewed only by the eye or by some optical instrument which can converge the light rays. The position of a virtual image can be found by focusing on it with a telescope, and then finding the distance for which the telescope has been focused.

Place a small lab telescope on the optical bench in place of the screen, as shown in Figure 25. Use any brightly illuminated point (such as a pencil) for the object, and adjust the telescope eyepiece so that the crosshairs are clearly in focus.

Looking through the telescope and through the diverging lens at the object, focus the telescope until you clearly see both the crosshairs and the image. If you move your head sideways and back again, the image and the crosshairs should appear to move together. If they do not, there is what we call *parallax.* The adjustment is not correct until there is *no* parallax. Temporarily remove the lens and look through the telescope, but do not readjust the focus. The object as seen through the telescope should now be out of focus. If you still see the object clearly, the lens was not properly aligned, and it will be necessary to replace the lens, and realign and

Figure 25.

refocus the telescope. When the object is out of focus with the diverging lens removed, you are ready to slowly move the object away from the telescope until it is again clearly in focus and there is no parallax between the object and the crosshairs. This new position of the object corresponds to the image position when the lens is in place. That is, the telescope stayed focused on the same spot, both when the object was at that spot and when its image was there.

You are now ready to make measurements using the diverging lens.

Use the appropriate work sheets at the end of this module. Write answers to questions, and complete the tables on those sheets as you do the experiment.

1. Record the position of the lens on the bench.

2. Turn the bench toward a distant object. With the telescope on the bench, and its crosshairs in focus, obtain a clear view of the image of the distant object through the lens. Remove the lens and check for misalignment. With the lens still removed, place a marker (such as a pencil point) on the optical bench in the view

of the telescope. Without refocusing the telescope, slide the marker along the bench until the marker is clearly in focus with the crosshairs. Check to see that there is no parallax, as before. You have now found the position of the image of a distant object for this lens and, therefore, you have also found the position of one of its focal planes. Record this position.

3. Replace the lens at the center of the bench, turn it around, and find and record the position of the other focal plane.

Note that the image of a distant object is formed on the *same* side of the diverging lens as the object; therefore, for measurements, the focal plane on that same side is associated with the image. The focal plane on the side of the diverging lens opposite the object will be associated with the object measurements.

Using the equation $f = H_i/\alpha$ you will make a direct measurement of the image height H_i and the angular size of the object, α. To achieve a sufficiently precise determination of f, we need to measure the angle α with an instrument which has a precision of at least $0.1°$. A surveyor's transit, a spectrometer table with its accompanying telescope, or any other precision angle-measuring device will do.

Place the angle-measuring device next to the lens and at the same distance from the object as the lens. If you measure the image height with a cathetometer, the object must be aligned vertically; therefore, if you use a spectrometer table to measure α, you must turn the table on its side so that it will measure angles in a vertical plane as shown on the right side of Figure 26.

4. Measure the angle between two reference points located one above the other in the distant scene. Choose these points so that the angle you measure is between $2°$ and $5°$. Convert your degree measurement to radians. (Radians = degree $\times$ 3.14/180.) Record this angle.

Figure 26.

The image height can be measured with a cathetometer (a telescope that can travel on an accurately ruled vertical column). View the distant scene with the cathetometer telescope *through the lens* as shown on the left side of Figure 26. It may be necessary to focus the telescope to get a clear picture of the image through the lens. Note the position of the lens on the bench. You should now verify that when you view through the telescope, you are actually viewing through the lens as well, by temporarily removing the lens. The distant scene viewed through the telescope should then be out of focus. If the scene remains in focus, you were not viewing through the lens, and it will be necessary to realign the apparatus.

5. With the lens in place again, accurately measure the vertical distance between the same two reference points used in the angular measurement. Read this distance as the image height, H_i.

6. Calculate f for this lens using the equation $f = H_i/\alpha$. Record this value of f.

Next you will investigate object and image distances for a diverging lens.

7. Place an object anywhere on the optical bench. Place the lab telescope on the

opposite side of the lens. Record the position of the object and find and record the position of its virtual image using the telescope, as you did previously. With the lens in the same position, find and record two more object positions and their corresponding image positions.

8. Find three values of object distance, x, by calculating the distance between the three object positions and the focal plane on the *opposite side* of the lens. Find the corresponding values of image distance, x', by computing the distances between the three image positions and the focal plane on the *same side* of the lens as the object.

9. Compute the product, xx', for each of the three pairs of values. Do your data support the hypothesis that the relationship $xx' = f^2$ is valid for a diverging lens?

PRINCIPAL RAY DIAGRAMS FOR DIVERGING LENSES

In Experiment B-2, you found that a diverging lens has focal points and focal lengths. Also the object and image distances were found to obey the same law as a converging lens, $xx' = f^2$. Therefore, it is reasonable to expect the method of principal ray diagrams to be applicable to diverging lenses.

As shown in Figure 27, three rays are chosen for ray diagrams with diverging lenses. One ray is parallel to the axis, one ray goes to the first principal point, and the third ray is toward the object focal point (which is on the opposite side of the lens from the object for a diverging lens). Precisely as with a converging lens, all rays now transfer from the first principal plane to the second principal plane parallel to the axis. The ray originally parallel to the axis cannot go through the image focal point (which is now on the same side as the object), it must be made to bend so that it *appears* to have come from that focal point. The ray through the principal points again emerges parallel to its original direction. The ray originally aimed toward the object focal point emerges parallel to the axis.

As you may have already noticed, the ray-tracing procedure for diverging lenses is quite similar to that for converging lenses. The major difference is that the object and image focal points have interchanged positions. From Figure 27 you can see that the rays emerging from the lens never actually intersect. If they are all extended backward, they appear to have come from a single point which is the image of the corresponding point on the object.

In general, real and virtual images differ in just this respect. The rays actually *pass through* the points on the real image but only *appear* to have come from the points of a virtual image.

Example Problem. Locate the image by means of a principal ray diagram, and find its height, for an object of height 2 cm, using a diverging lens which has a focal length of 4 cm and whose principal planes are separated by 1 cm. Take the object distance to be 12 cm.

Solution. It must be remembered that for a diverging lens the object and image focal points are on opposite sides of the lens (or principal planes) from the object and image,

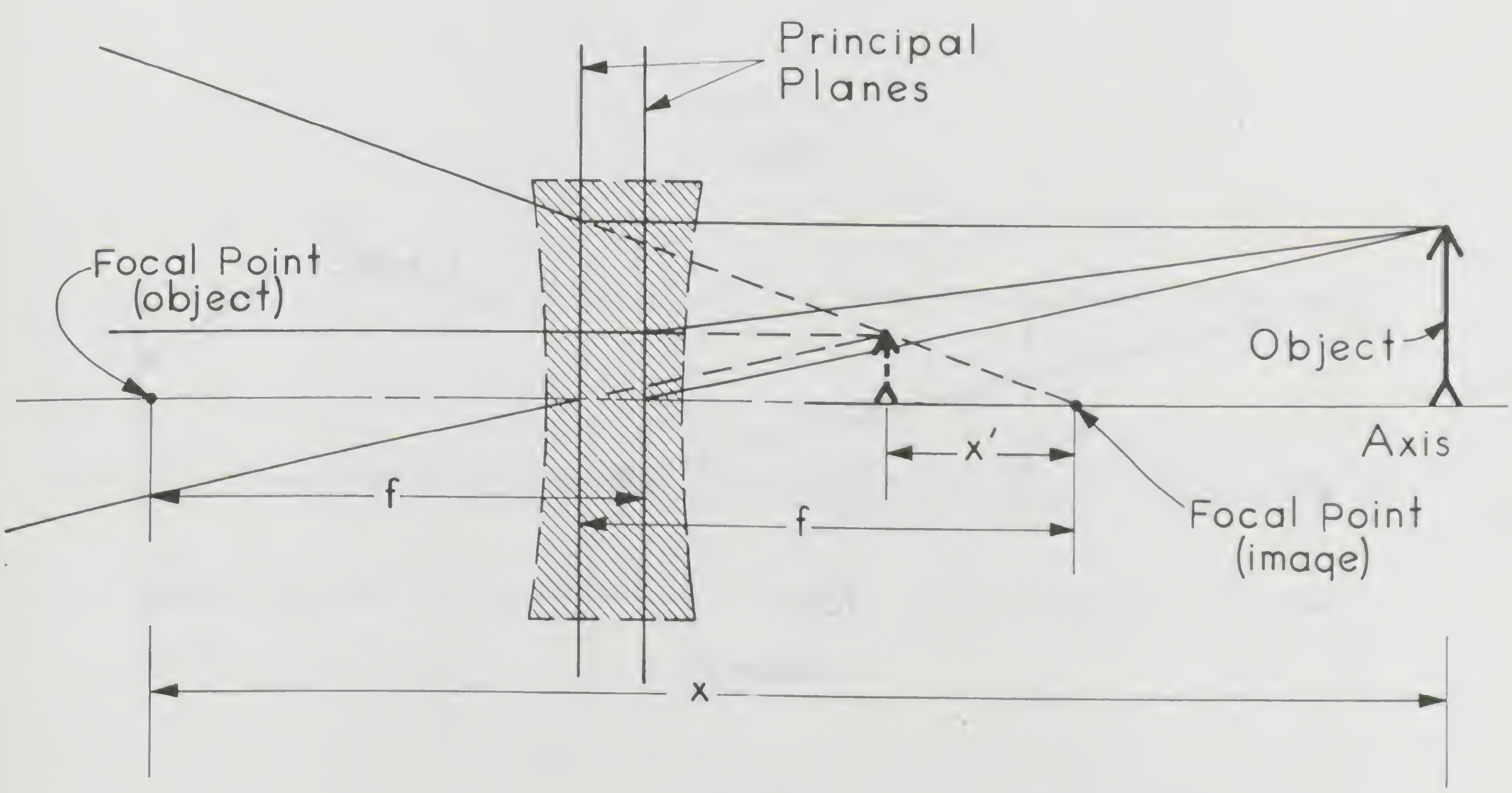

Figure 27.

respectively. Figure 28 shows the axis and the two principal planes drawn as before.

If the object is placed to the right of these principal planes (the lens), then the object focal point is located to the left of the principal planes and the focal length is measured (in centimeters) from the plane which is nearest the object, as shown in Figure 29.

The image focal point will be located similarly, but it is to the right of the principal planes and is measured from the left plane as shown in Figure 30.

We will use a 2-cm high arrow for the object in this example, as shown in Figure 31, and it must be drawn 12 cm to the right of the object focal point.

As shown in Figure 32, the first ray is drawn from the top of the object, parallel to the axis, to the first principal plane.

Again, this ray, as do all rays, must pass parallel to the axis on going from one principal plane to the other. At the second

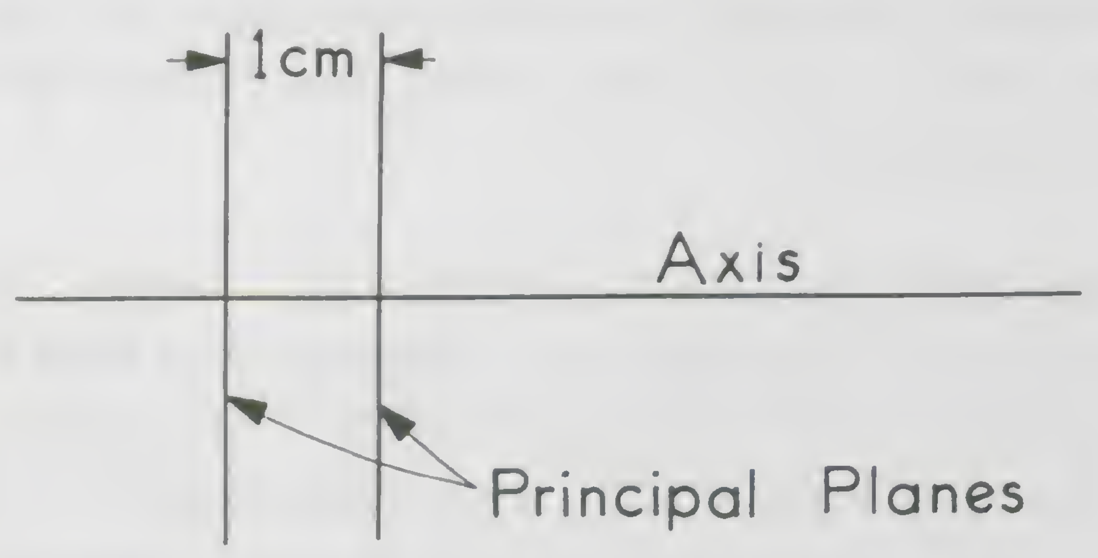

Figure 28.

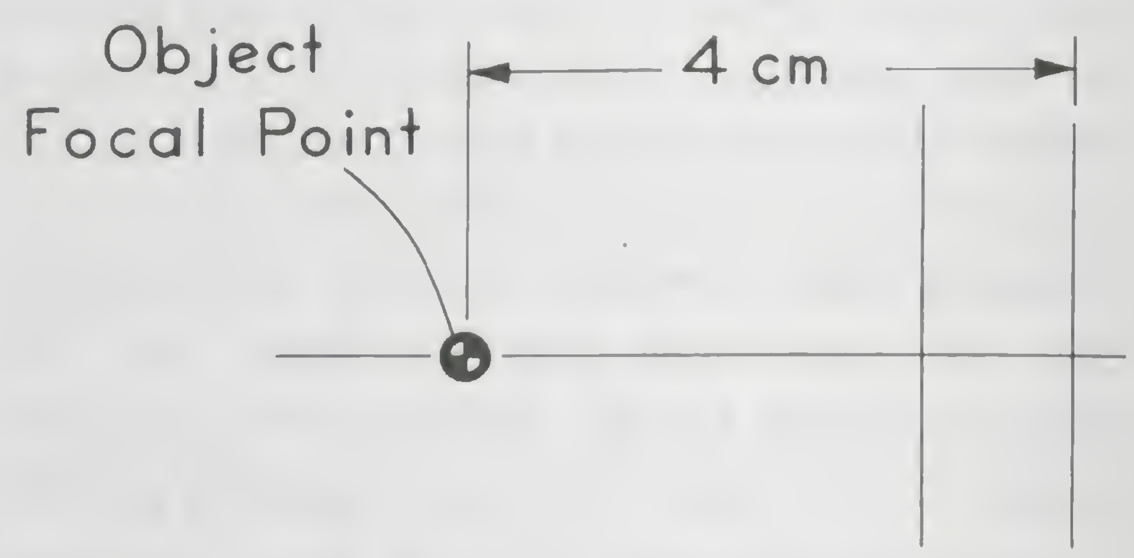

Figure 29.

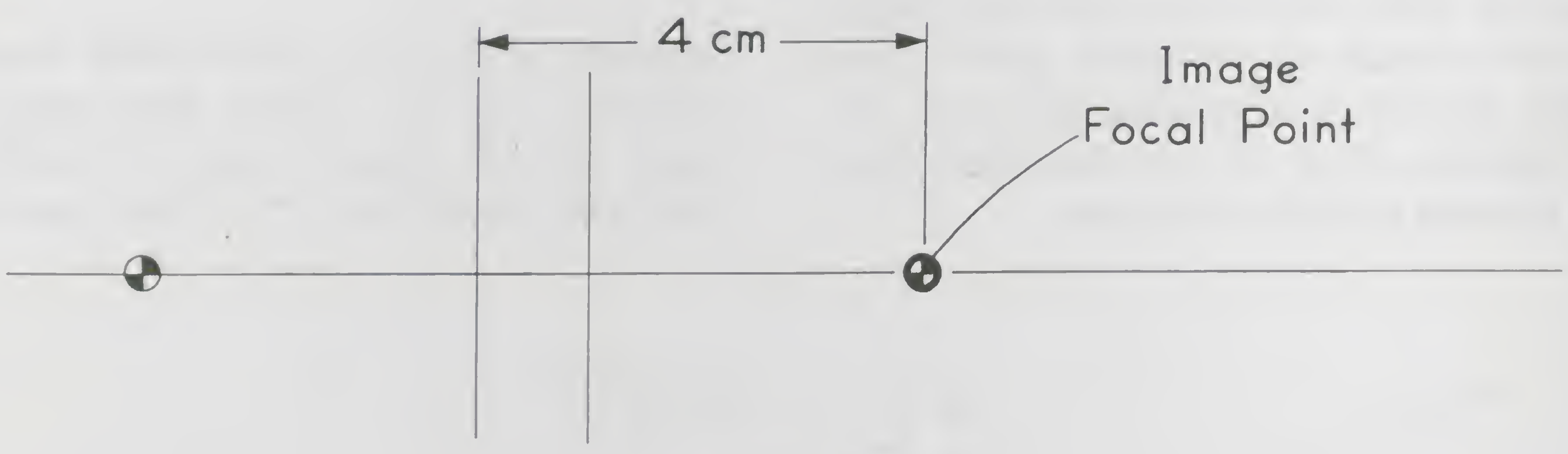

Figure 30.

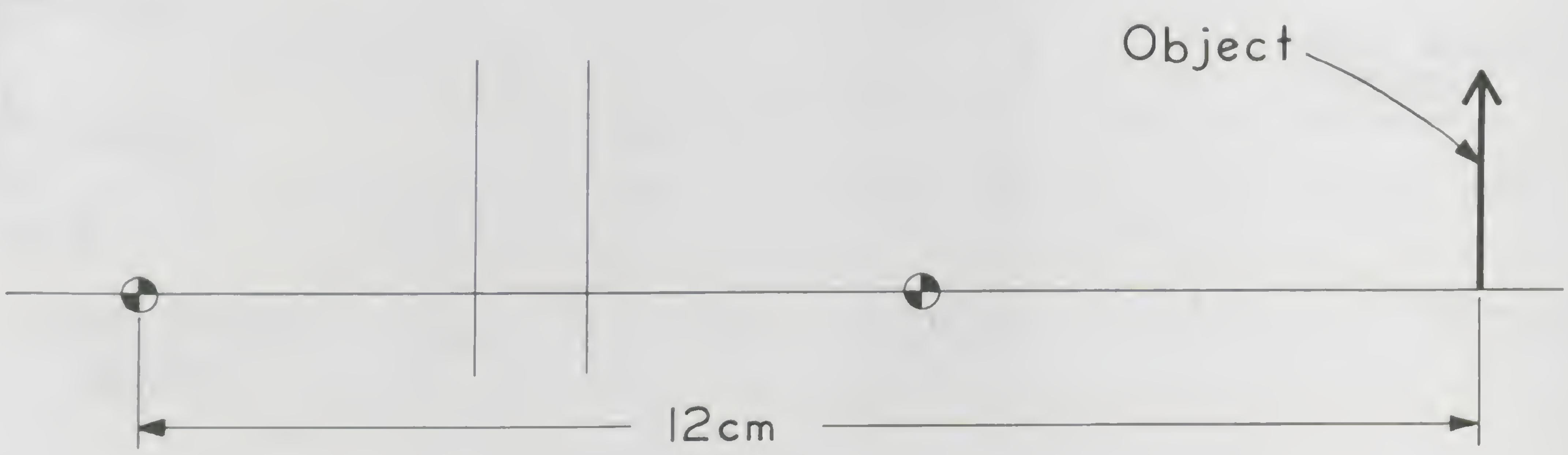

Figure 31.

principal plane the ray bends, just as in the case of the converging lens, toward the image focal point. However, in this case the image focal point is to the right of the principal planes. The ray cannot go through this focal point; therefore, it is bent away from the axis, as shown in Figure 33, so that it *appears* to have come from the image focal point.

The second ray begins at the same spot on the object and goes to the first principal point, as shown in Figure 34.

This ray travels along the axis between principal planes and then bends, as shown in Figure 35, away from the axis at the same angle that it made with the axis on the opposite side of the principal planes.

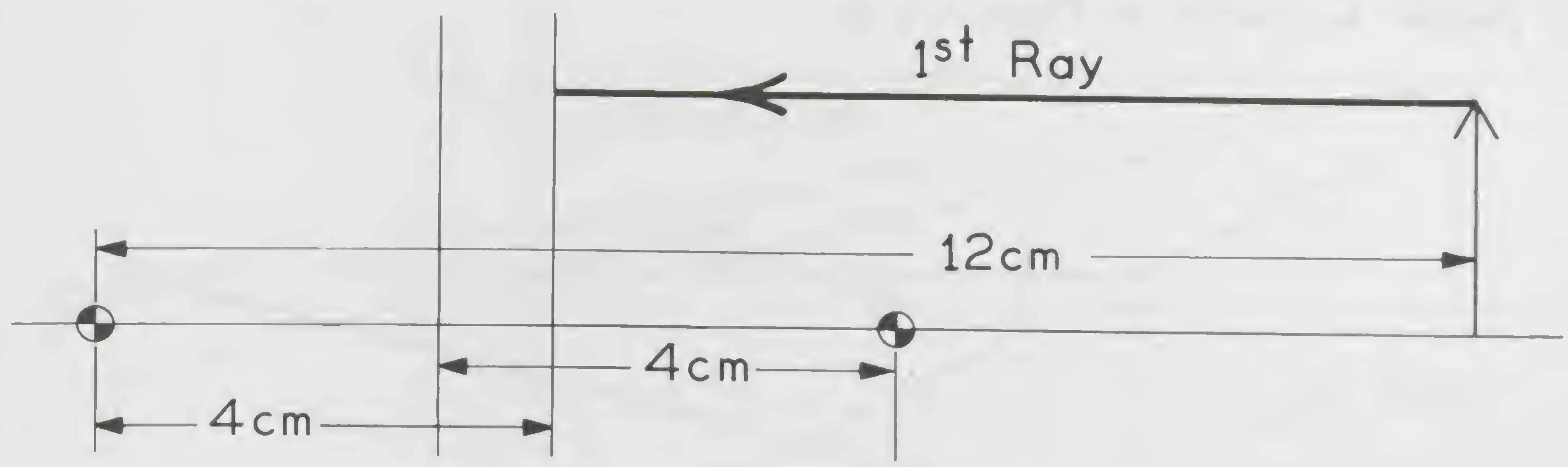

Figure 32.

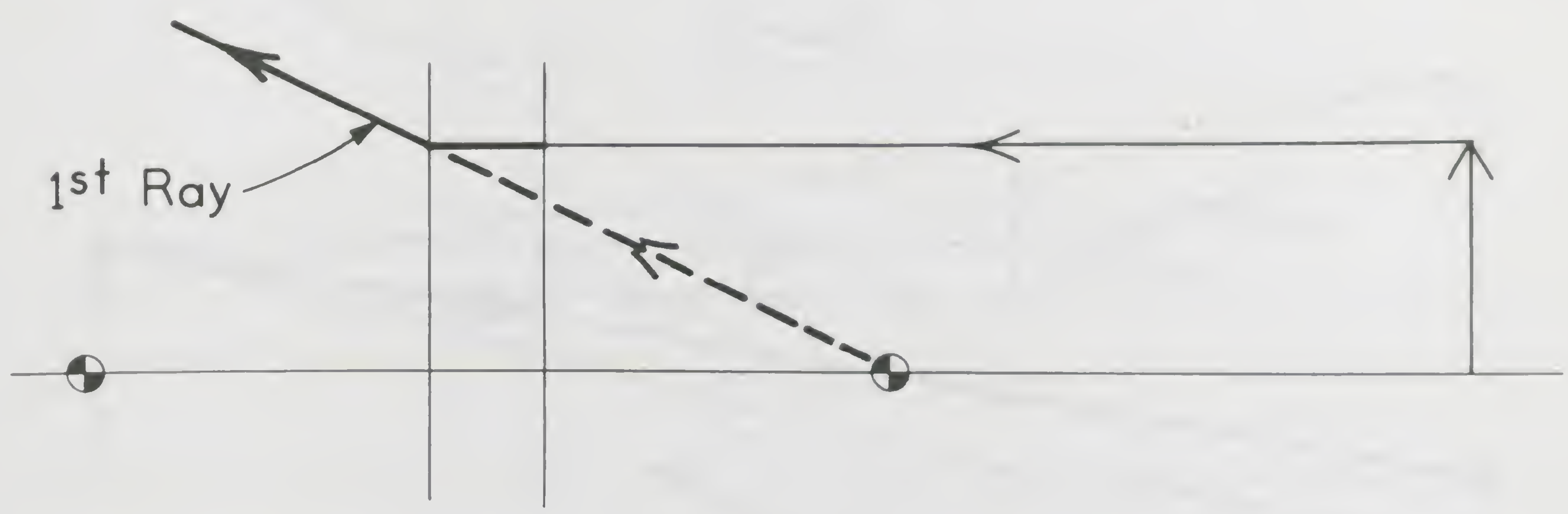

Figure 33.

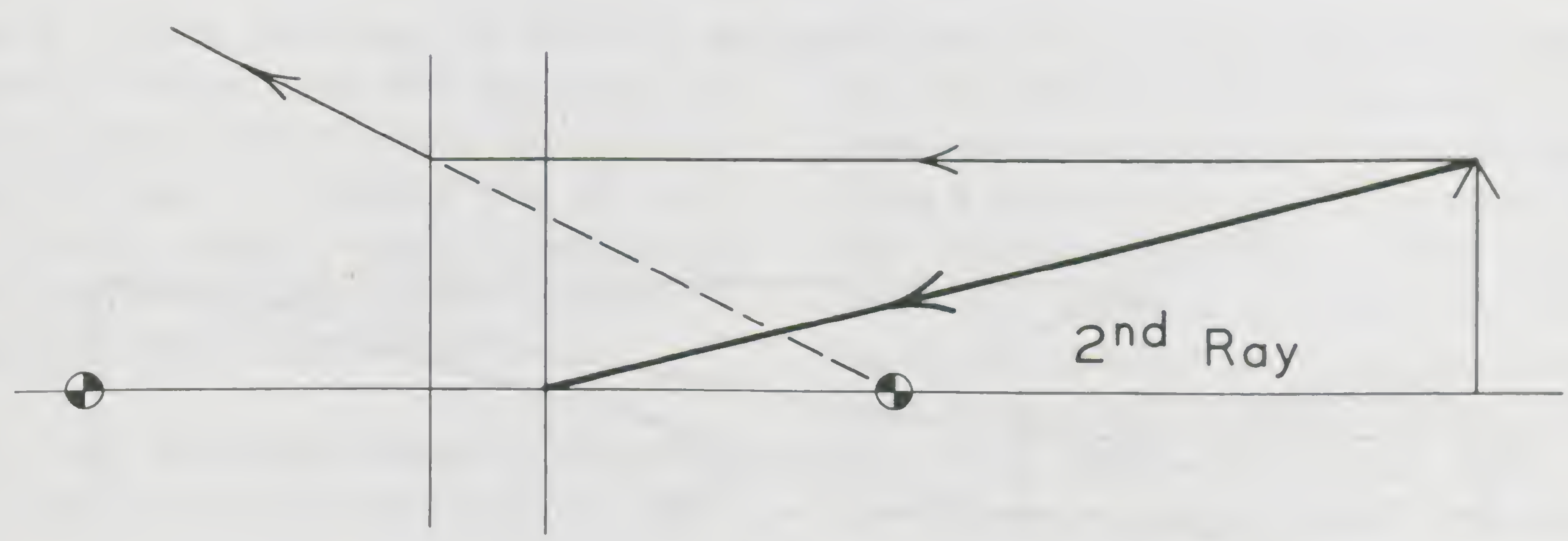

Figure 34.

In the case of the converging lens, the third ray went through the object focal point. For the diverging lens we also draw this ray toward the object focal point. Since this focal point is on the opposite side of the lens or principal planes, however, it must be drawn toward that point, as shown in Figure 36, but will be bent when it reaches the first principal plane.

This ray travels parallel to the axis between principal planes. As shown in Figure 37, it then travels parallel to the axis after leaving the second principal plane.

Now that all the rays are constructed it can be seen that the three rays diverge as they leave the lens and they will never come together to form an image. However, these rays *appear* to come right from a point to the right of the principal planes, and it is here that the virtual image of the tip of the arrow appears.

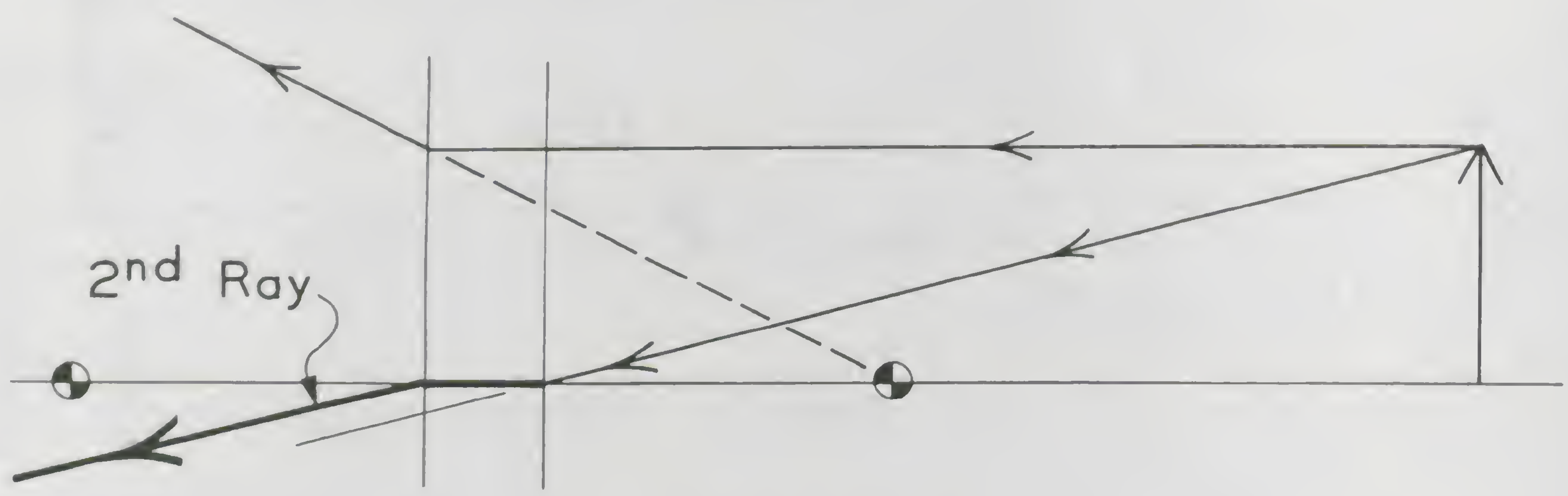

Figure 35.

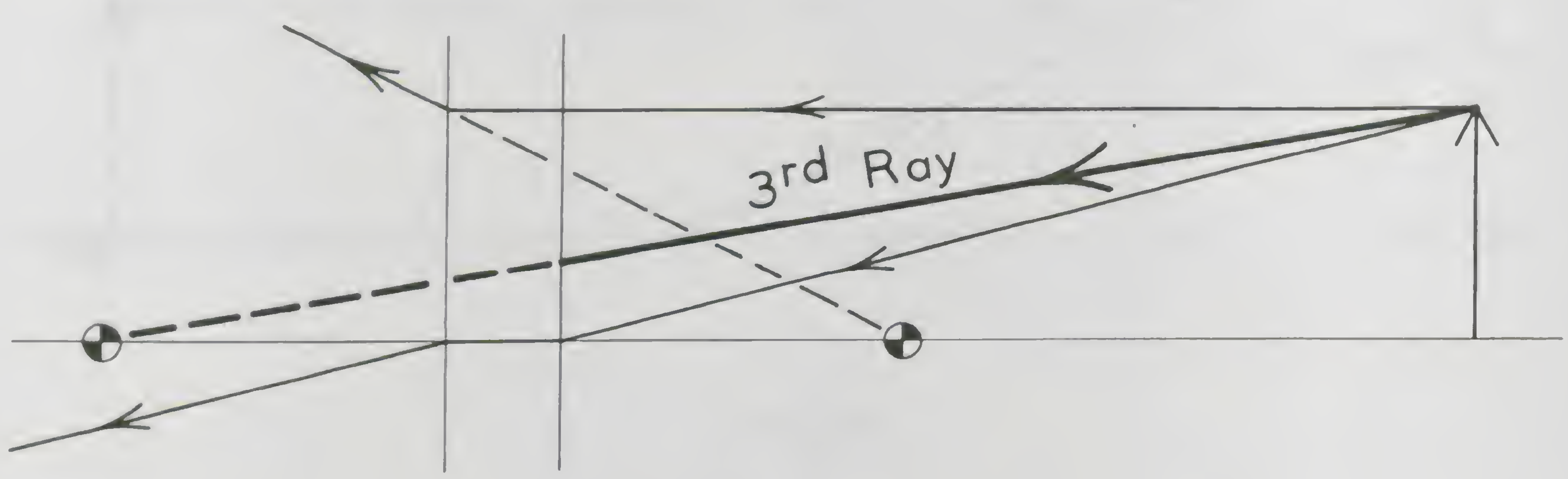

Figure 36.

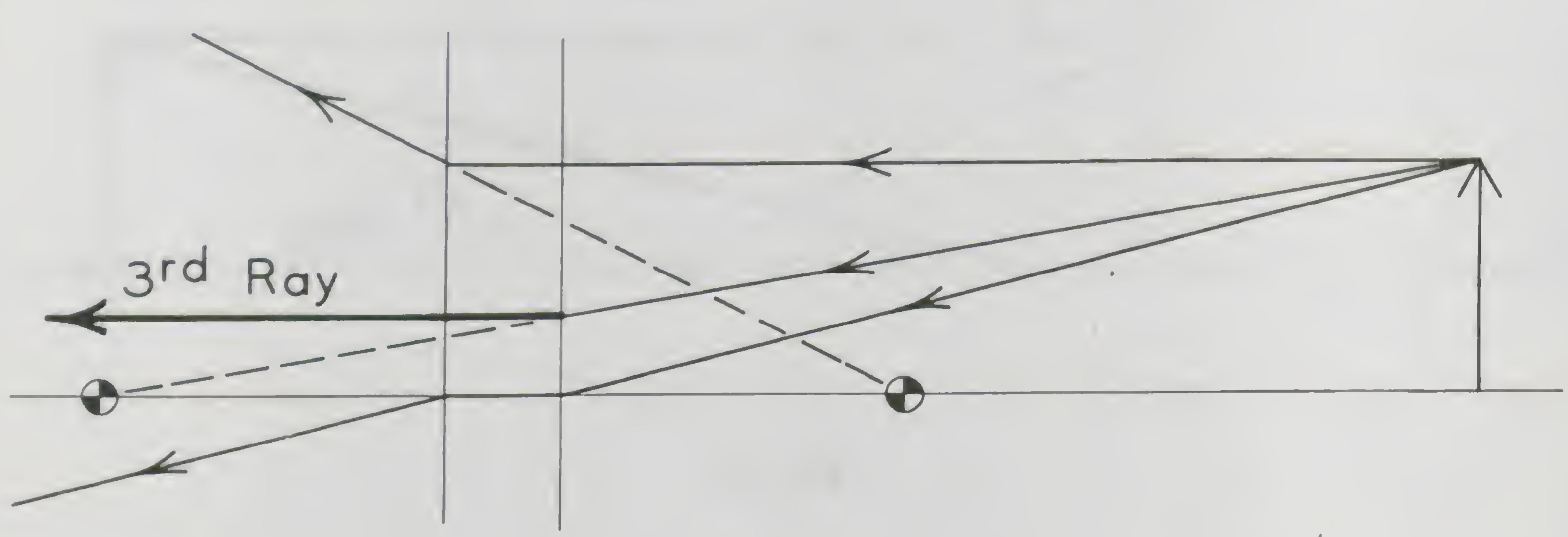

Figure 37.

To find the virtual image point, as shown in Figure 38, all three rays may be extended straight back from their intersections with the second principal plane. These three extensions come together at a point to form the tip of the virtual image.

As shown in Figure 39, the base of the image is on the lens axis below the tip. The position of the image, which was defined as its distance from the image focal point, may now be measured. (It must be remembered that the image focal point is to the right of the lens.) The height may also be measured. The results, from Figure 38, are 1.3 cm for the distance and 0.7 cm for the height.

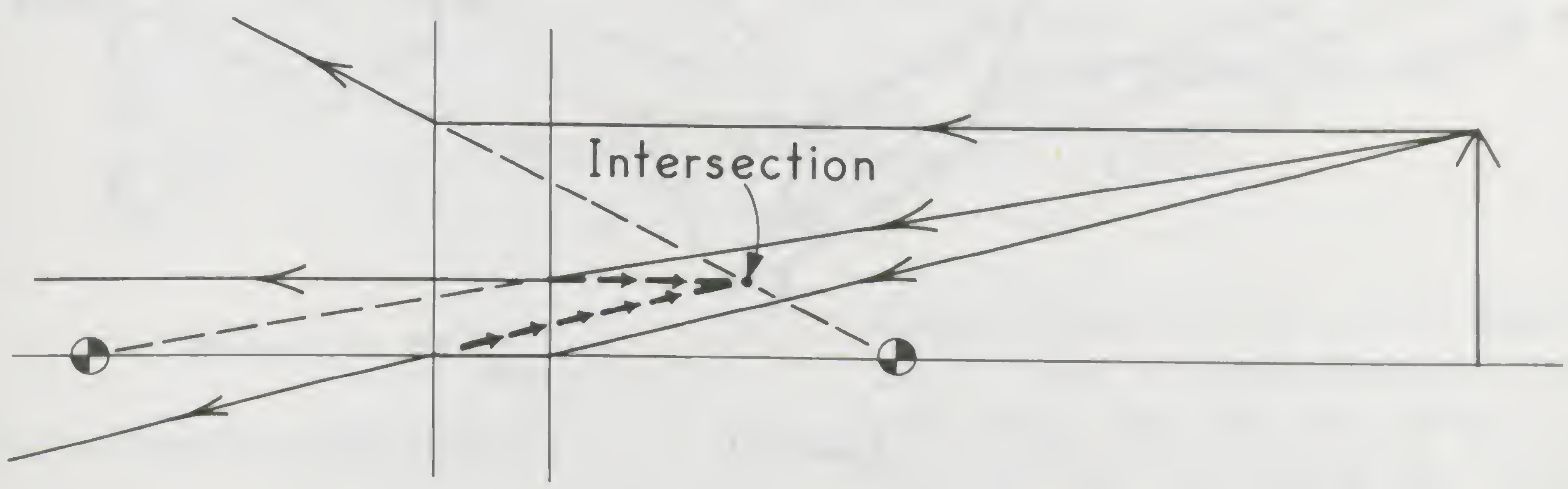

Figure 38.

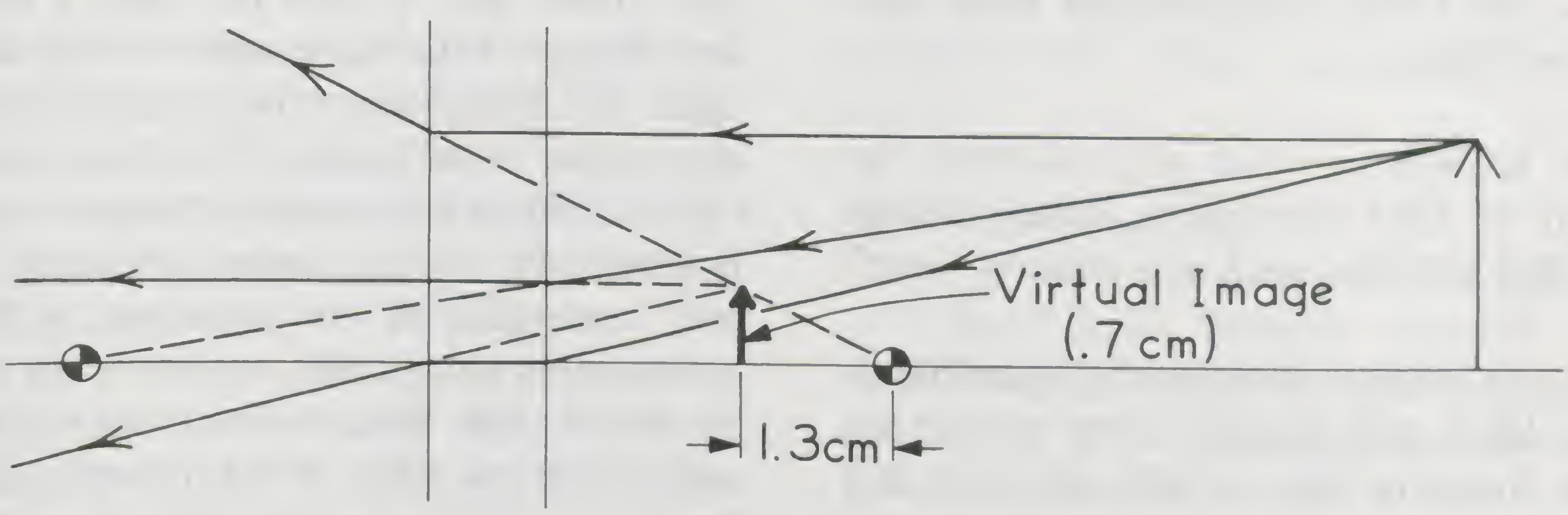

Figure 39.

Problem 9. Locate the image by means of a principal ray diagram, and find its height for an object whose height is 3 cm. The object is observed through a diverging lens of focal length 5 cm, whose principal planes are separated by 2 cm. The object is located 15 cm from the object focal point.

Why Do the Binoculars Need a Diverging Lens?

In the seventeenth century, Sir Isaac Newton noticed small rings of color surrounding the real image of the sun which was produced by a simple converging lens. That, and other similar observations, led Newton to investigate these color effects more carefully. Using a prism, as shown in Figure 40, he was able to demonstrate that "white" light contained the colors of the rainbow. The prism bends different colors by different amounts. (This is called *dispersion.)* To prove to himself that white light did contain these colors, Newton used another prism, with additional apparatus, to bring the colors together again to form white light.

The explanation for dispersion is that the index of refraction of glass is slightly

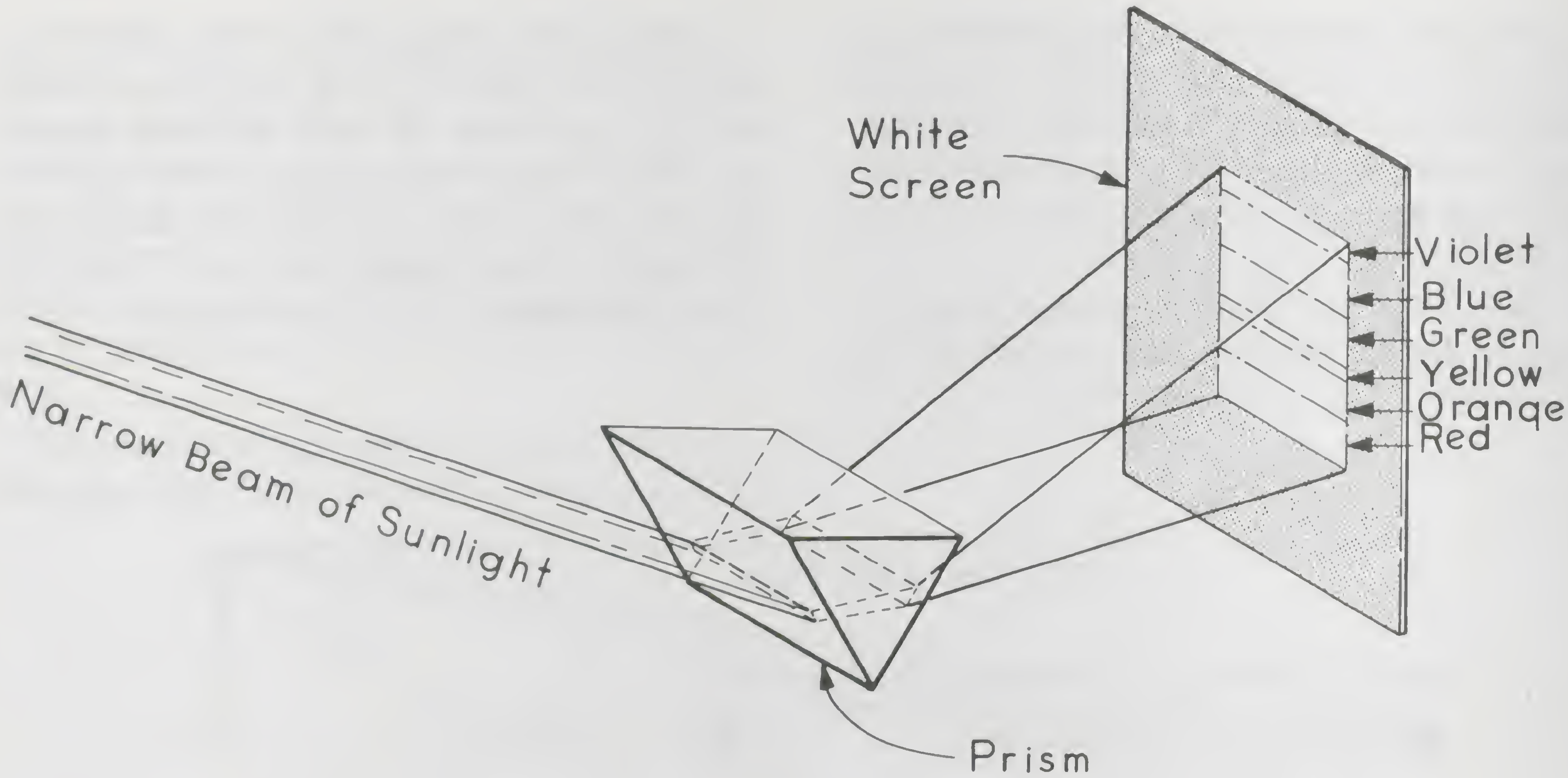

Figure 40.

different for different colors of light. For example, the index is higher for blue light than for red light.

Question 8. Why do we not observe the dispersion of light through a piece of glass with parallel surfaces, such as a window pane?

Because objects are usually illuminated by white light, the objective lens of a monocular is focusing light of all colors of the rainbow. But, since the light at the violet end of the spectrum is converged more than the red light at the other end of the spectrum the different colors of the object each has its image focused at a slightly different location. There are two results of dispersion in converging lens.

1. The focal points are slightly different for the different colors.
2. Because the ratio of image height to object height depends on focal length, the magnification is also different for different colors from the same object.

These effects are called *chromatic aberration* (the first is called *longitudinal chromatic aberration,* the second *lateral chromatic aberration*).

Just as a converging lens converges blue light more than it does red light, a diverging lens diverges blue light more than it does red light. By selecting a diverging lens having a high index of refraction, like dense flint glass (n = 1.73), whose index changes by 0.025 from the red to the blue end of the spectrum, and cementing it to a converging lens of crown glass (n = 1.50), whose index changes by only 0.008 from the red end to the blue end of the spectrum, the dispersion produced by the converging lens is canceled out by the dispersion produced by a diverging lens. As shown in Figure 41, the net result is to correct completely for chromatic aberration.

SUMMARY

In this section of the module, you have learned the law of reflection:

a. The reflected ray lies in the plane of the incident ray and the *normal* to the reflecting surface.
b. The angle of incidence equals the angle of reflection, the angles being measured from the normal to the surface.

You have also learned Snell's Law of Refraction:

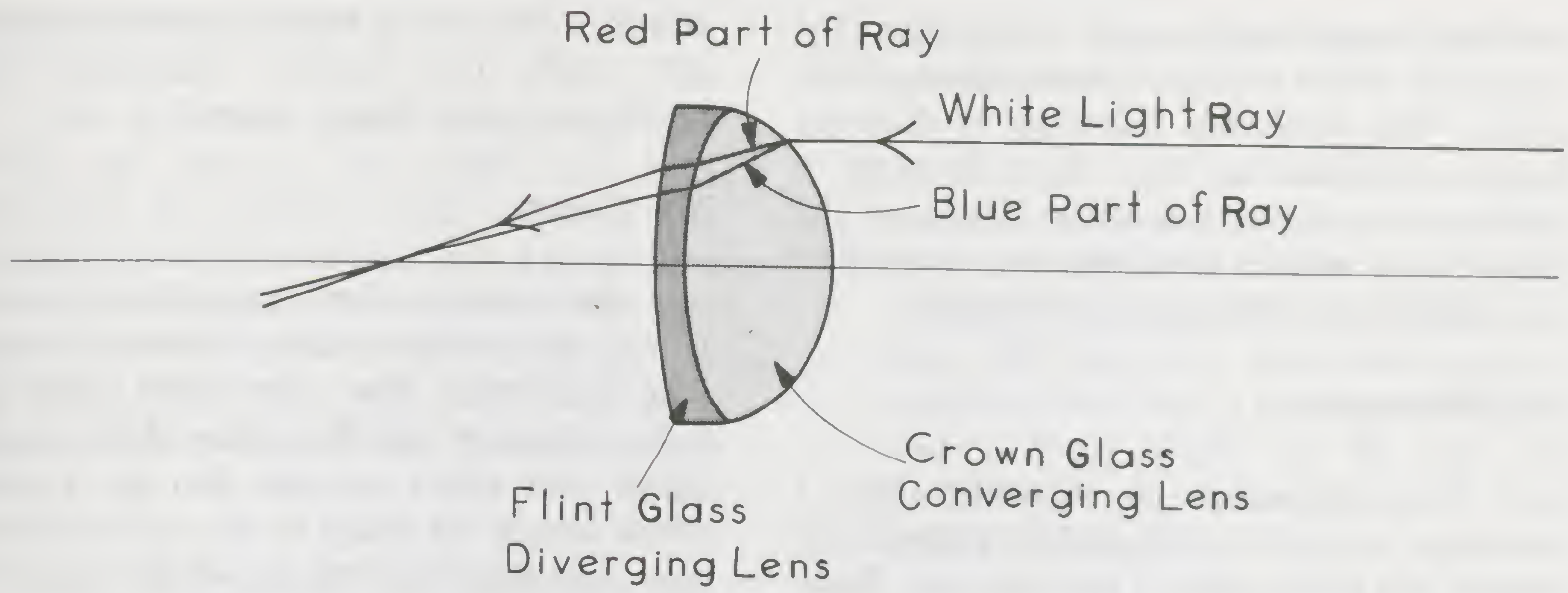

Figure 41.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

You learned that the relationships

$$xx' = f^2 \text{ and } H_i/H_o = f/x$$

apply to the compound lens systems, as well as to thin lenses. The same relationships were found to apply to a diverging lens.

Because the relationships just listed apply to real lenses or lens systems, the method of principal ray diagrams also applies. You learned that to use principal ray diagrams with real lenses you needed only to locate *principal* planes, and to make measurements from these principal planes. (The method of principal ray diagrams is otherwise the same as appears in *The Camera* module for single lenses.)

Finally, you learned that the correct combination of a diverging lens and converging lens can be used to eliminate chromatic aberration.

APPLICATIONS OF PRINCIPLES TO OTHER DEVICES

The Nearsighted Human Eye

For a nearsighted eye, rays of light from a distant object are focused in front of the retina by the crystalline lens of the eye. Thus the eye lens converges these rays too much. Distant objects cannot be seen clearly; objects can be seen at closer distances than can be seen clearly with the normal eye. To correct this problem, a *diverging lens* must be used either in eyeglasses or contact lenses. The combination of the diverging lens and the eye lens then focuses light rays from a distant object on the retina as required for distant vision.

The Magnifying Glass

You have already learned that a magnifying glass is just a converging lens, with the object to be viewed placed between the focal point and the lens.

The Farsighted Human Eye

For a farsighted eye, light rays from objects which are nearby cannot be focused on the retina. They would, instead, be focused behind the retina. The eye lens cannot converge these rays sufficiently. To see clearly with such an eye, objects would have to be held at a distance and could not be seen distinctly if they are close. To correct for this problem, a converging lens must be used, either in eyeglasses or contact lenses.

The Telescope

As you have seen, the refracting telescope consists simply of a converging lens (the objective), which has a long focal length

to form a real image, and a magnifier (the eyepiece) which produces an enlarged, virtual image. The *reflecting telescope* is different from a refractor in that the real image is produced by a spherical mirror. But then the magnifier is used in the same way to produce an enlarged virtual image of the object.

The Microscope

The microscope is basically like a telescope, except that the object is placed just outside the focal point of the objective. Since the object is very small, the real image, although very much larger than the object, is still small. The eyepiece (magnifier) then produces a larger virtual image.

The Camera

Most high-quality cameras have a compound lens system which consists of several individual lenses. These compound lenses have a focal plane at the film plane of the camera. Single lens reflex cameras also use a mirror which directs the image to the eye for framing and focusing, then flips out of the way when the picture is taken.

GOALS FOR SECTION C

The following goals state what you should be able to do after you have completed this section of the module. The example which follows each goal is a test item which fits the goal. When you can correctly respond to any item like the one given, you will know that you have met that goal. Answers appear immediately following these goals.

1. *Goal:* Understand the definition of angular magnification.

   Item: Suppose that you view a distant tower through a telescope. The tower is actually 100 ft tall and is 5000 ft away. The virtual image of the tower formed by the eyepiece is 2 cm high and 25 cm from your eye. What are the values in radians of the angle subtended by the object for an unaided eye and by the virtual image for the aided eye? What is the angular magnification of this telescope?

2. *Goal:* Be able to calculate the magnification of a telescope or monocular.

   Item: A telescope has an eyepiece which has a focal length of 2 cm. The objective has a focal length of 30 cm. What is the angular magnification of the telescope?

3. *Goal:* Know how to determine from the standard specification of a monocular its magnification and the diameters of its objective lens and exit pupil.

   Item: An 8 X 40 monocular is used to observe a distant scene. What is the magnification of this monocular? What are the diameters of the objective lens and the exit pupil?

4. *Goal:* Understand how the brightness of the image seen through a telescope or monocular depends upon the instrument specifications.

   Items: A 7 X 40 monocular is used to look at the surface of the moon. How does the brightness of the image of the moon produced by this monocular compare with that of a telescope having an objective lens with a diameter of 50 mm and a focal length of 40 cm? The eyepiece of the telescope has a focal length of 5 cm.

5. *Goal:* Be able to calculate the critical angle for total internal reflection in a substance.

   Item: A large block of transparent material having an index of refraction of 1.39 has a ray of light incident to the boundary from the inside. What is the value of the critical angle?

6. *Goal:* Know how to select binoculars under certain conditions of use.

   Item: Which of the following binoculars would you use for
   a. viewing objects from hand-held binoculars while standing on a 30-ft cruiser on open ocean in bright daylight, and
   b. viewing the moon's surface at night with the binoculars mounted on a clock-driven, rigid support:

Binoculars A	6 X 30
Binoculars B	8 X 50
Binoculars C	7 X 35

   In terms of the optics of the binoculars, and of your eye, explain how you made your selections.

Answers to the Items Accompanying the Preceding Goals

1. ϕ_u = 100 ft/5000 ft = 1/50

   ϕ_a = 2 cm/25 cm = 2/25

$$M = \frac{\phi_a}{\phi_u} = (2/25)/(1/50) = 4$$

2. $M = f_o/f_e$ = 30 cm/2 cm = 15

3. M = 8; Diameter of Objective = 40 mm
Diameter of exit pupil = D/M = 40 mm/8 = 5 mm

4. $$\frac{B_1}{B_2} = \left(\frac{D_1/M_1}{D_2/M_2}\right)^2$$

We have D_1 = 40 mm and M_1 = 7.

For the telescope, $M = f_o/f_e$ = 40/5 = 8, and D = 50 mm

$$\frac{B_1}{B_2} = \left(\frac{40/7}{50/8}\right)^2 = \left(\frac{8}{50} \times \frac{40}{7}\right)^2$$

$$= (32/35)^2 = .84$$

The image seen through the binoculars is 84% as bright as the image through the telescope.

5. $\sin \theta_c = 1/n = 1/1.39 = 0.72$

$\theta_c = 46°$

6. a. Binoculars A. You need the smallest magnification because it is hard to hold the binoculars still on the cruisier.
b. Binoculars B. You can use the largest exit pupil and magnification, since your eye is dark-adapted and the binoculars are held still.

SECTION C

Magnification of Binoculars and Porro Prisms

INTRODUCTION

In this section, we will develop a theoretical equation for the magnification of a telescope (or a monocular) and a theoretical equation for the diameter of the exit pupil of such an instrument. For each case, you will do an experiment to compare theoretical predictions with what is observed experimentally. Finally, the basic principle of the porro prism system will be studied and applied.

Ray Diagram for Simple Telescopes

As you learned in Experiment A-1, when viewing a distant object the distance between the objective lens of a telescope and the eyepiece is very nearly the sum of the focal lengths of the two lenses. This means that, for a distant object, the real image of the object, which is produced by the objective lens, is located nearly at the objective lens focal point. This focal point is located between the two lenses. The real image is then just inside the focal point of the eyepiece. For most real telescopes, this distance is so small that, for practical purposes, we can say that the image is at the focal point of the eyepiece. We therefore are saying that the two focal points coincide. This means that the lenses are separated by a distance very nearly equal to the sum of the focal lengths of the two lenses.

Problem 10. Suppose that you make a telescope out of two thin converging lenses. One lens has a focal length of 50 cm and the other lens has a focal length of 2 cm.

a. Which lens would you use as the objective lens and which would you use as the eyepiece?
b. Either by using principal ray diagrams or by using the equation $xx' = f^2$, locate the real image of a distant object.
c. The telescope uses the *image* of the objective as the *object* for the eyepiece. Suppose that you adjust the position of the eyepiece so that the virtual image of the eyepiece is located 200 cm away from the eyepiece. Use either ray diagrams or equations to determine the distance between the two lenses.

If we construct two principal rays, one from the top and one from the bottom of a distant object through the center of the objective lens, the rays form an image at the focal point of the objective, as shown in Figure 42.

From the real image formed by the objective to the eyepiece, two other rays, one from the top of the image and one from the bottom of the image, are drawn through the center of the eyepiece.

THE ANGULAR MAGNIFICATION OF A TELESCOPE

You should remember that the rays of Figure 42 are present along with many others, and we have just picked out these particular ones for convenience. You should note that the two rays from the real image which pass through the eyepiece are not the same ones shown passing through the objective; there is nothing at the real image that could bend the rays. These four rays drawn in Figure 42 define the angle subtended by the object, ϕ_u (unaided), and the angle subtended by the final virtual image, ϕ_a (aided eye).

The angular magnification is defined as the ratio of these two angles; if M stands for angular magnification, then

$$M = \phi_a / \phi_u \qquad (5)$$

Since both angles are usually small, we can see in Figure 42 that the following approximate relations for the angles are valid:

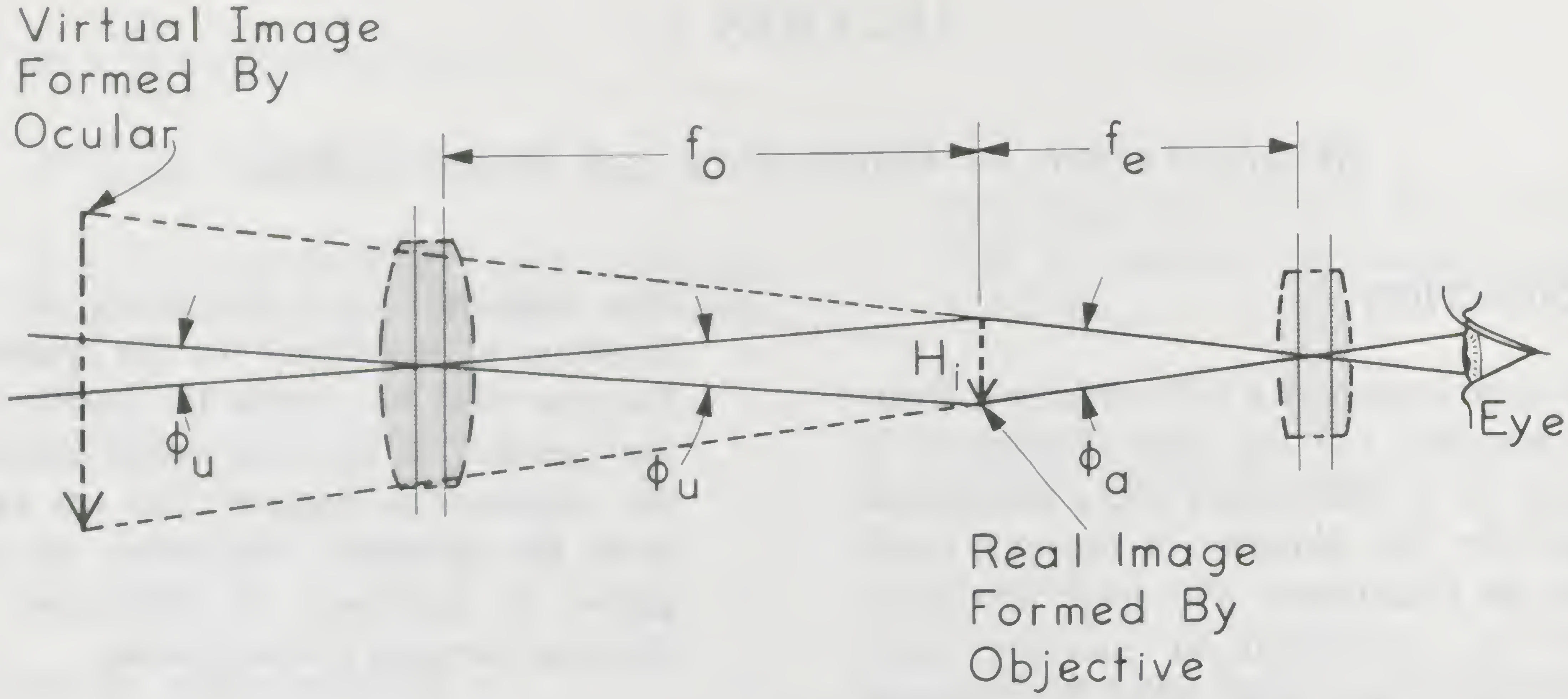

Figure 42.

$$\phi_a = H_i/f_e \tag{6}$$

$$\phi_u = H_i/f_o \tag{7}$$

When these values are substituted into Equation (5) for the respective angles, the result is

$$M = f_o/f_e \tag{8}$$

The angular magnification depends on the focal lengths of the two lenses used for objective and eyepiece. For high magnification, you need a long focal length objective and short focal length eyepiece, which confirms the results found in Experiment A-2. Does the actual angular magnification of a telescope really follow this simple equation? You can test this theory by doing the next experiment.

EXPERIMENT C-1. Angular Magnification and Focal Lengths

In this experiment you will compare theoretical predictions for the magnification of telescopes with the actual magnification observed by experiment. In effect, you will test the theoretical equation, $M = f_o/f_e$, to determine how well it agrees with the real world.

Take out the work sheets at the end of this module. Write answers to questions, and complete the tables on those sheets as you do the experiment.

1. Using a converging lens, a screen, and an optical bench, form a distinct object. Use the simplification (thin lenses) that the focal length is then the distance from the lens to the screen. Record this distance as the lens focal length. Mark this lens with a grease pencil so that you can later identify it.

   To estimate the error in this value of focal length, move the screen slightly toward the lens until the image just begins to lose its sharpness. Note this screen position. Next move the screen away from the lens until the image again just begins to lose its distinctness. Note this position of the screen. Half the difference of these two values of screen position is a good estimate for Δf_o, the error in your focal length measurement.

   Record these values and repeat this procedure for each of the other four lenses.

2. Use a short focal length lens as an eyepiece of the telescope. The focal length of the lens you have has been measured so accurately and precisely that you may assume, for this experiment, that the error in its measurement is negligible. Record the value of this focal length, as given by your teacher.

3. You will now calculate theoretical values for different values of objective focal length. Since you will make these calculations for an eyepiece whose focal length we are assuming we know perfectly, these calculated values do not depend upon your experiment. To decide upon the range of objective focal lengths which will be used, select the lowest value of objective focal length to be about the same as the lowest value measured in step 1, and the highest value close to the upper limit measured. Select four or five somewhat evenly spaced values of f_o between these limits, and substitute these values for f_o into the equation $M = f_o/f_e$. Use the value of f_e already given to you, and calculate theoretical values of magnification M. Record these values of M and f_o.

4. Label coordinate axes on graph paper and plot magnification, M, on the vertical axis and objective focal length, f_o, on the horizontal axis. Use the upper and lower limits of M and f_o determined in step 3. This is your theoretical prediction of the magnification of a telescope having an eyepiece of focal length, f_o, for any value of objective lens focal length within the limits of the graph.

5. Place the eyepiece on the optical bench. Select one of the other lenses to use as an objective. Place this lens on the optical bench in front of the eyepiece. Adjust the distance between the lenses to obtain the largest clear image of a distant object, when you view through this telescope.

6. With one eye, look through the telescope you have constructed toward distant, evenly spaced lines, as you have done before. At the same time, look at the same evenly spaced lines with the other unaided eye. As you learned earlier, the magnification can be determined by counting how many such spaces you see with your unaided eye which fit into

some number of spaces seen through the telescope. The magnification is just the quotient of these two numbers.

Devise some method for estimating the error in this experimental value for magnification. Record these values in the table. Repeat this procedure for the other four objective lenses. Use the same techniques as in step 1 to estimate focal length errors, Δf_o.

7. On the same graph paper as you used to show your theoretical predictions, plot a graph of experimentally observed magnifications on the vertical axis and measured focal lengths on the horizontal axis. Find the slope of this graph.

   To show the error estimates, ΔM and Δf_o, construct rectangles centered on your data points. (The width of an error rectangle will be $2\Delta f_o$ and the height of a rectangle will be $2\Delta M$.) Construct two more lines, one a line with maximum slope which will stay within the error rectangles, the other, a line with minimum slope which will stay within the error rectangles. Find slopes of these lines and record them. Now you can estimate the error in the slope as one half the difference in the two values.

8. Since the theoretical graph is that of the equation $M = f_o/f_e$, the slope has the value $1/f_e$. The reason for this is that we can write the equation as $M = (1/f_e)f_o$, and the slope of an equation is the coefficient of the variable plotted on the horizontal axis.

   Does the theoretical line have a slope which falls between the limits of your experimentally determined slope? Does the theoretical equation adequately predict the magnification of a telescope?

TELESCOPE MAGNIFICATION

In Experiment C-1, you found that the theoretical equation

$$M = f_o / f_e$$

predicts the magnification of a telescope having an objective with focal length f_o and eyepiece with focal length f_e.

It is possible to relate the magnification of a telescope to the diameters of the entrance and exit pupils of the telescope.

Entrance and Exit Pupils of a Telescope

Since the monocular is essentially a telescope with inverting porro prisms, the ray-tracing diagram for an astronomical telescope will be most helpful in understanding how the instrument works. Figure 43 shows such a diagram with two rays, 1 and 2 coming from the top of a distant object and rays 3 and 4 from the bottom of the same object.

The object is so far away that rays from a single point on it that enter the lens are essentially parallel. However, the object is large enough so that there is a noticeable angle between the rays from the top and the rays from the bottom. (As a matter of fact, the "object" could be the whole field of view of the instrument.) The rays from a single point on the object also emerge from the eyepiece parallel to each other, although the rays from the top of the object (1 and 2) now make a larger angle with the rays from the bottom of the object (3 and 4). The fact that the rays from a single point emerge parallel means that the eye will see them coming from a very distant object: we say that the final virtual image is at infinity. The larger angle between the rays from the top and bottom of the object is what is meant by angular magnification, and the final virtual image will appear larger than the original object when viewed without the instrument.

Figure 43 also shows that the exit pupil is a real, inverted image of the objective lens: rays 1 and 3 diverge from a point at the top of the objective lens and converge again at the bottom of the exit pupil after passing through the eyepiece. This means that the eyepiece forms a real image of the top of the objective at the bottom of the exit pupil. Similarly, rays 2 and 4 converge at the top of the exit pupil to form the image of the bottom of the objective.

Figure 43 also displays the property of the exit pupil which allowed you to find it in Experiment A-1: it is the position where the emerging cone of light narrows to its smallest diameter. The smallest circle of light is therefore the exit pupil.

Relating Magnification to Pupil Diameters

Figure 44 shows two rays from the center of a distant object. The two rays are almost parallel when they reach the objective lens. The diameter of the exit pupil is labeled

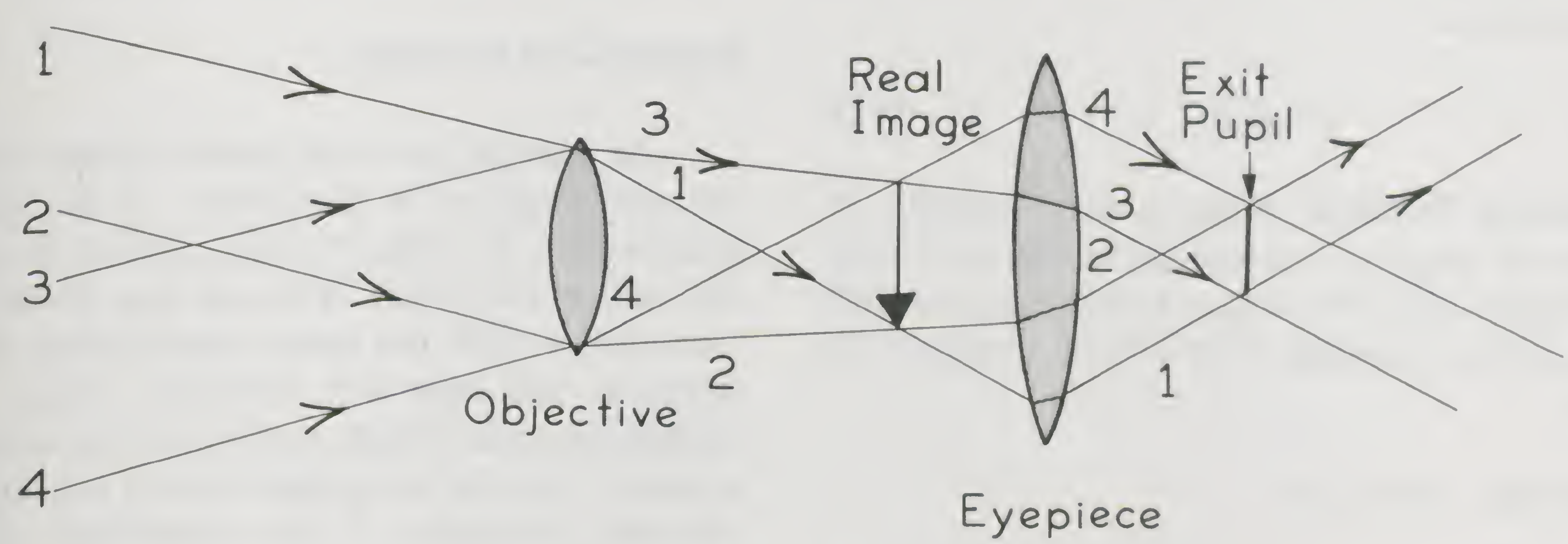

Figure 43.

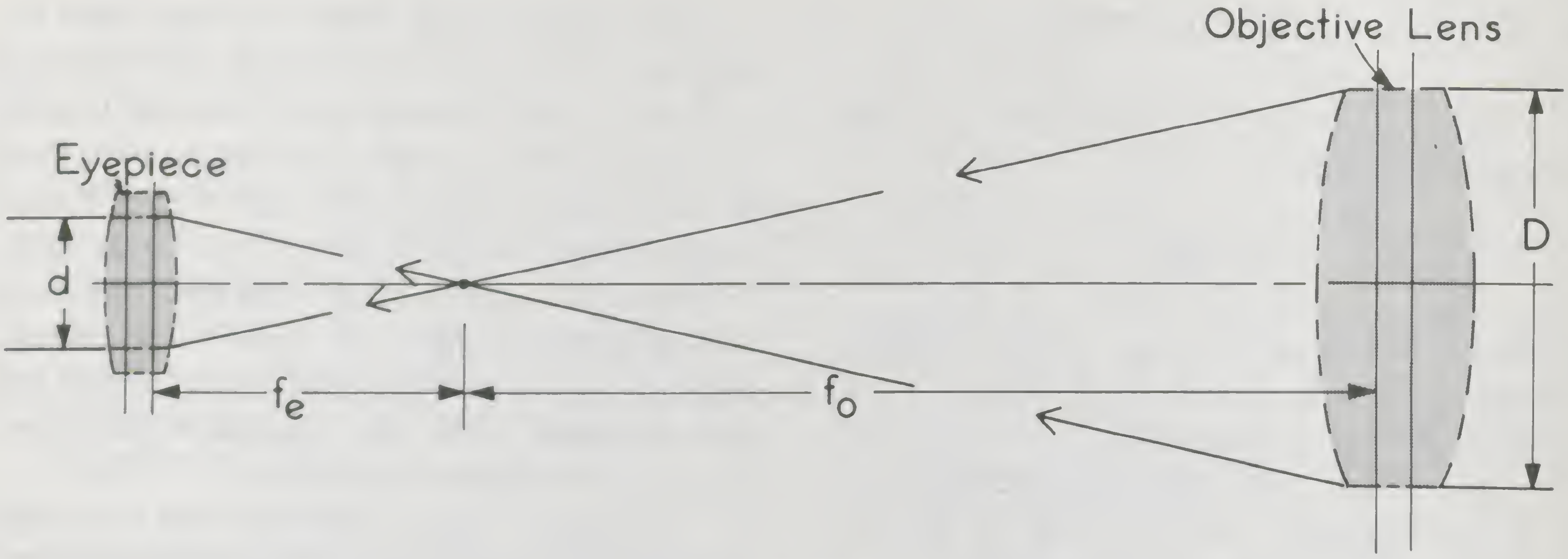

Figure 44.

d, although its exact location is not evident from Figure 44 alone. It would be where the two parallel bundles of light cross behind the eyepiece in Figure 43. The diameter of the entrance pupil is shown as *D*. The rays and diameters form similar triangles from which we can set up the proportion

$$d/f_e = D/f_o \tag{9}$$

or

$$d = Df_e/f_o \tag{10}$$

But we know from Equation (8) that the ratio of the focal lengths is equal to the magnification. We then have

$$D/d = M \tag{11}$$

The ratio of the pupil diameters is equal to the magnification. We also can write this equation as

$$d = D/M \tag{12}$$

Example Problem. What is the diameter of the exit pupil of a telescope having an 8.0-cm diameter objective with a focal length of 200 cm and an eyepiece with a focal length of 10 cm?

Solution. Given are

$$D = 8.0 \text{ cm}, f_o = 200 \text{ cm}, \text{ and } f_e = 10 \text{ cm}$$

Substituting these values into Equation (10), we have

$$d = \frac{8.0 \text{ cm} \times 10 \text{ cm}}{200 \text{ cm}}$$

Simplifying,

$$d = 80/200 \text{ cm}$$

or

$$d = 0.4 \text{ cm}$$

Problem 11. What is the diameter of the exit pupil of a telescope having a 6.0-cm diameter objective of focal length 300 cm and an eyepiece of focal length 10 cm?

Problem 12. What is the diameter of the exit pupil of a monocular having an objective diameter of 50 mm and a magnification of 8?

Brightness of the Image

As long as your eye pupil is larger than the exit pupil of a monocular, all the light gathered by the objective lens enters your eye and strikes the retina. If we use two different monoculars with the same magnification but different objective lens diameters, then the brightness of the image formed on your retina is greater for the monocular having the larger objective diameter. If the monoculars have different magnifications but objectives of the same diameter, then the amount of light

entering your eye is the same for each instrument. However, for the monocular of higher magnification, the image on your retina is larger. Thus the light is spread out more, and is therefore less bright.

The brightness of an image is inversely proportional to the square of the image height. But the height of the image produced on your retina is directly proportional to the magnification of the monocular or telescope you are using: $H_i \propto M$. Therefore, the brightness of the image on your retina is inversely proportional to the square of the magnification of the instrument being used: $B \propto 1/M^2$. Also, for a fixed magnification, the brightness of the image on your retina is directly proportional to the square of the diameter of the objective lens: $B \propto D^2$. (These properties are valid only when the entrance pupil of your eye is larger than the exit pupil of the binoculars.)

Combining these relationships, we can write the proportion

$$B \propto D^2/M^2$$

We have just seen that the exit pupil diameter, d, is given by

$$d = D/M$$

Thus, the brightness follows the relation

$$B \propto d^2$$

or, in terms of focal length and the objective diameter

$$B \propto \left(\frac{f_e D}{f_o}\right)^2$$

As we mentioned before, the image seen through a telescope or binoculars will be brightest when the exit pupil is the largest which will completely enter your eye. Of course, when the exit pupil is larger than your eye pupil, the light does not all get into your eye, so there is no advantage in having the additional light.

Problem 13. How much brighter is the image on your retina from a distant object as seen through a 7 X 35 monocular than from a 10 X 40 monocular?

EXPERIMENT C-2. Dependence of Magnification of a Monocular on Pupil Diameters

Testing the Theory

We have seen that the theory of telescopes (or of monoculars) predicts that the magnification, M, can be calculated from the equation

$$M = D/d$$

where D is the diameter of the objective (the entrance pupil) and d is the diameter of the exit pupil.

In this experiment you will be using a monocular with the magnification given in the standard specification. This value of magnification is the theoretically predicted value for this instrument and you will test to see if it matches experimentally.

Take out the work sheets at the end of this module. Write answers to questions, and complete the tables on those sheets as you do the experiment.

1. Record the value of M predicted by the manufacturer of your monocular.

2. Mount your monocular on an optical bench. Measure the diameter of the objective lens and estimate the error in this measurement. This is the entrance pupil. Place the ground glass plate behind the eyepiece and aim the objective toward a well-lighted area. Move the ground glass toward or away from the eyepiece until you have a well-focused circle of light. This is the exit pupil. Measure its diameter, and estimate the measurement error. Place the adjustable diaphragm in front of the objective as close as possible to the lens. This will enable you to vary the size of the entrance pupil. Change the entrance pupil, measure its new diameter (D), and estimate the error (ΔD). Measure the corresponding exit pupil diameter (d) and estimate its error (Δd). Repeat this procedure for at least five diameters of the entrance pupil.

3. One method of estimating errors in a result of a division of two numbers, is to add the percentage errors of the two numbers being divided. This sum is a good estimate of the percentage error in the calculated quality For each of the entrance pupil diameters, the percentage error is just

   $$\text{Percentage error} = \Delta D \times 100$$

   For each of the sets of measurements in step 2, calculate the values of D/d, the percentage error in D, the percentage error in d, and the sum of these percentages. Record these results in the table.

4. Find the average of your five values of $M = D/d$ and the average percentage error in this quantity. Using this average percentage, calculate the error estimate for the experimentally determined magnification. Is the theoretically predicted value of magnification for this monocular within the range of values observed (average value of M ±error estimate)?

PORRO PRISMS

In Experiment B-1 you saw that when a light ray strikes a boundary between two transparent media, it splits into both a refracted and a reflected ray. In other words, part of the light is reflected and part is refracted and passes through. For example, when looking at a window from the outside, you can see a reflection of outside light, whereas an observer inside sees light from the outside which has been transmitted through the window. As you saw in Experiment B-1, when a light ray in a medium of higher refractive index strikes a boundary with a medium of lower refractive index at a large enough angle, all of the light is reflected. Thus, a ray traveling in glass which is surrounded by air may all be reflected back into the glass if the angle of incidence is sufficiently large. This phenomenon is called *total internal reflection.*

Total internal reflection occurs in this situation because the light ray is always bent away from the normal as shown in Figure 45, where the angle of incidence, θ_1, is too small for total internal reflection.

This bending insures that the angle of incidence, θ_1, will be less than the angle of refraction, θ_2. Since θ_2 increases when θ_1 increases, θ_2 will become 90° when θ_1 is still less than 90°: for some angle of incidence the refracted ray will just be grazing the surface as

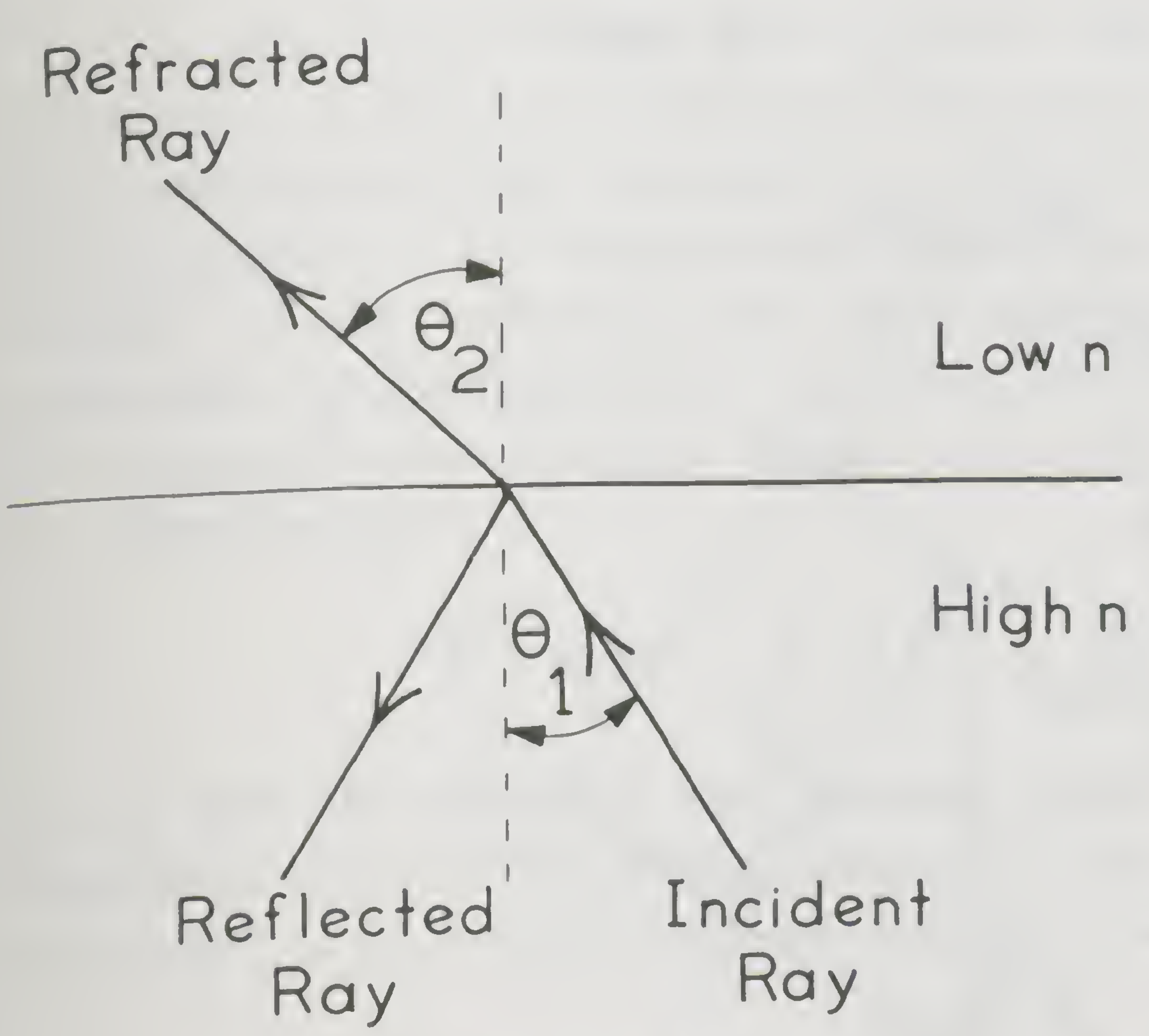

Figure 45.

shown in Figure 46. The angle of incidence when the angle of refraction is 90° is called the *critical angle.* In Figure 46, θ_c is the critical angle. Any angle of incidence larger than the critical angle gives rise to total internal reflection. The value of the critical angle depends on the index of refraction of the material in which the ray is traveling.

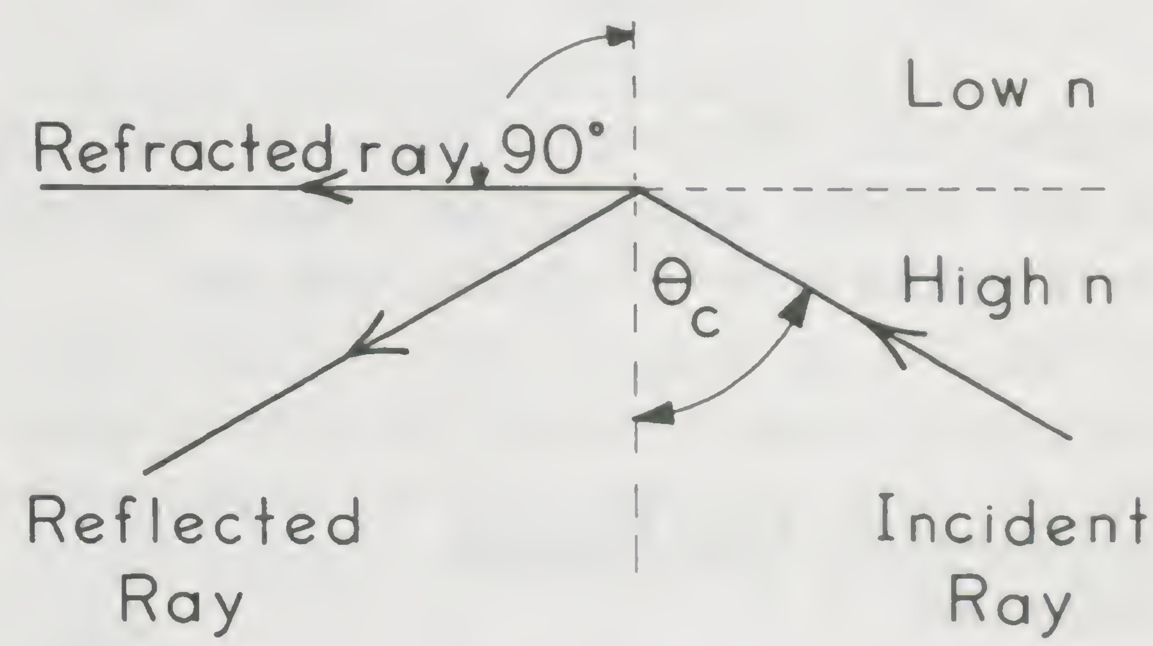

Figure 46.

Question 9. Why isn't there total internal reflection when light travels from a medium with a lower index of refraction into one with a larger index?

The critical angle for most types of glass in air is less than 45°. Therefore a light ray that is incident on a glass-air interface at 45° will be totally internally reflected. The porro prisms in a monocular make use of this effect. We need only apply the law of reflection to see that the ray enters and leaves one prism as shown in Figure 47.

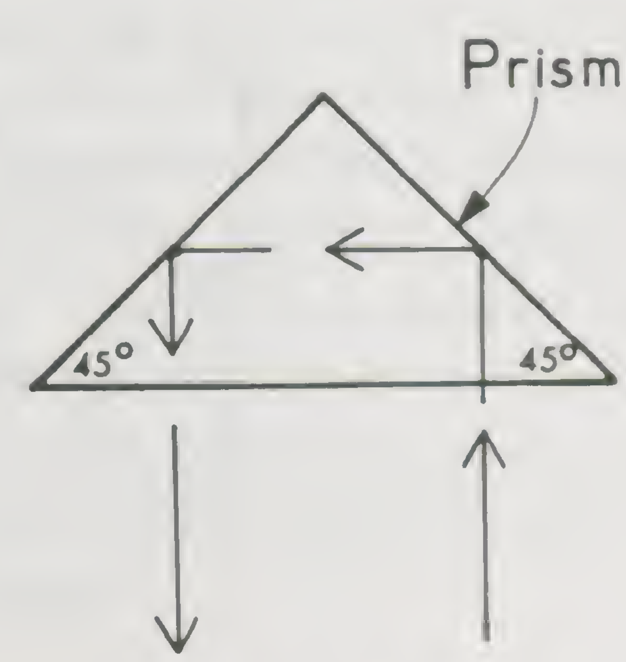

Figure 47.

The pair of prisms working together invert the image as shown in Figure 48. You can also see why the prisms result in a shortening of the monocular body. The path of the light is folded up into a more compact configuration, even though its total length is the same as that of the corresponding astronomical telescope.

Question 10. What would Figure 47 look like if the critical angle for the prism material were greater than 45°? Sketch this case.

Condition for Total Internal Reflection

As noted earlier, the distinguishing feature of monoculars or binoculars is the porro prism assembly which uses total internal reflection. We can derive a relationship for the critical angle in total internal reflection. You may recall that total internal reflection can occur only when the light is traveling from a medium of higher refractive index to one of lower index. Figure 49 shows a light ray incident on a boundary at the critical angle. We are assuming the medium with the lower index of refraction is air (or a vacuum), whose index is just about 1. The angle of incidence in this case is labeled θ_c, since it is just the critical angle. Then we can use Snell's Law (Equation 4b) to write

$$n \sin \theta_c = 1 \sin 90^\circ$$

But the sine of 90° has a value of 1. Therefore, we can solve for the sine of the critical angle

$$\sin \theta_c = 1/n \qquad (13)$$

Thus the larger the index of refraction, the smaller the critical angle.

Question 11. For light entering a prism at random angles, are you more likely to see total internal reflection (once the light is inside) if the material has a higher or lower refractive index? Why?

Example Problem. The index of refraction for many kinds of glass is about 1.5. What is the critical angle for this glass?

Solution. Given is just

$$n = 1.5$$

This value can be substituted in Equation (13)

$$\sin \theta_c = 1/1.5 = .667$$

Using the tables in the Appendix we find

$$\theta_c \approx 42^\circ$$

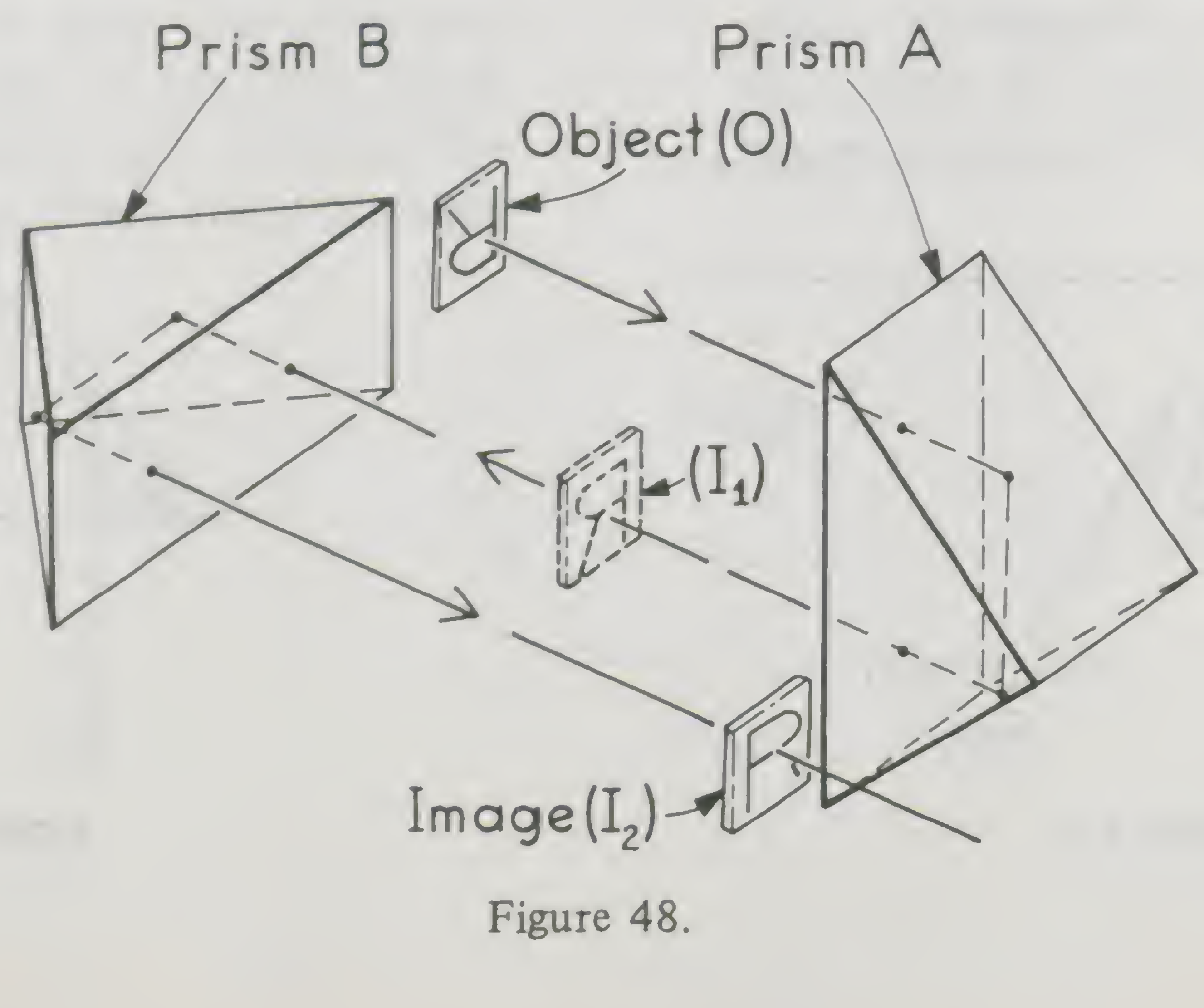

Figure 48.

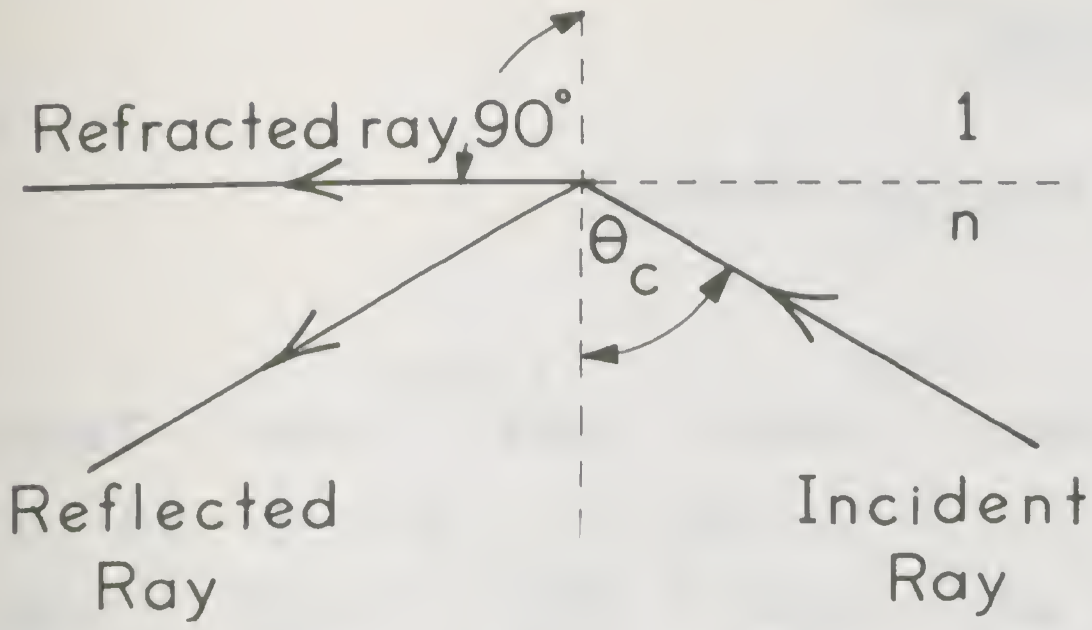

Figure 49.

This result means that a ray incident on a glass-air boundary will be totally internally reflected if the angle of incidence is greater than 42°, which explains why the 45°-45°-90° prisms in the monocular work. (See Figure 47.)

Problem 14. The index of refraction of diamond is 2.4. What is the critical angle in diamond?

Problem 15. If the critical angle for water is 49°,what is the index of refraction of water?

Problem 16. Why does a diamond sparkle more than glass of the same cut?

SUMMARY

In this section you have learned that there are a number of equations which relate the properties of the monocular as a whole to the properties of the components. The magnification of the monocular or binoculars (or telescope) is determined by focal lengths of the objective and eyepiece lenses, according to Equation (8)

$$M = f_o/f_e \quad (8)$$

Along with the objective lens diameter, these quantities then determine the diameter of the exit pupil, as expressed in Equation (12)

$$d = D/M \quad (12)$$

The final monocular image seen is upright because of a series of four total internal reflections in two porro prisms between the objective and eyepiece.

Total internal reflection occurs whenever the angle of incidence is greater than the critical angle. The critical angle is determined by the index of refraction of the prism material. According to Equation (13)

$$\sin \theta_c = 1/n \quad (13)$$

This equation tells us that the 45°-45°-90° glass prisms will effectively reflect all the light transmitted by the objective into the real image; almost no light is lost in these reflections.

BUYING BINOCULARS

Suppose you are going to buy binoculars. What would you look for to get a good one? Coated, achromatic lenses, Erfle eyepieces, firmly mounted porro prisms, and small weight are some things to look for. What about magnification and objective diameter? They depend upon the use to be made of the binoculars. High magnification reduces the field of view and makes it difficult to hold the binoculars firmly enough to keep the image still. A large diameter objective is of little value if the exit pupil is larger than the pupil of the eye under conditions of use. You should select binoculars on the basis of their use.

There are several topics concerning binoculars we have not considered. For example; anti-reflection coatings, field of view, secondary aberrations, astigmatism, wide angle systems, stereo vision, and special effects of prisms to correct aberrations. If you find this subject especially interesting, you may want to learn more about optics or the details of binocular design.

APPENDIX

Table of Trigonometric Functions (Natural)

Angle Degree	Angle Radian	Sine	Cosine	Tangent	Angle Degree	Angle Radian	Sine	Cosine	Tangent
0°	.000	0.000	1.000	0.000					
1°	.017	.017	1.000	.017	46°	0.803	0.719	0.695	1.036
2°	.035	.035	0.999	.035	47°	.820	.731	.682	1.072
3°	.052	.052	.999	.052	48°	.838	.743	.669	1.111
4°	.070	.070	.998	.070	49°	.855	.755	.656	1.150
5°	.087	.087	.996	.087	50°	.873	.766	.643	1.192
6°	.105	.104	.994	.105	51°	.890	.777	.629	1.235
7°	.122	.122	.992	.123	52°	.908	.788	.616	1.280
8°	.140	.139	.990	.140	53°	.925	.799	.602	1.327
9°	.157	.156	.988	.158	54°	.942	.809	.588	1.376
10°	.174	.174	.985	.176	55°	.960	.819	.574	1.428
11°	.192	.191	.982	.194	56°	.977	.829	.559	1.483
12°	.209	.208	.978	.212	57°	.995	.839	.545	1.540
13°	.227	.225	.974	.231	58°	1.012	.848	.530	1.600
14°	.244	.242	.970	.249	59°	1.030	.857	.515	1.664
15°	.262	.259	.966	.268	60°	1.047	.866	.500	1.732
16°	.279	.276	.961	.287	61°	1.065	.875	.485	1.804
17°	.297	.292	.956	.306	62°	1.082	.883	.470	1.881
18°	.314	.309	.951	.325	63°	1.100	.891	.454	1.963
19°	.332	.326	.946	.344	64°	1.117	.899	.438	2.050
20°	.349	.342	.940	.364	65°	1.134	.906	.423	2.145
21°	.366	.358	.934	.384	66°	1.152	.914	.407	2.246
22°	.384	.375	.927	.404	67°	1.169	.920	.391	2.356
23°	.401	.391	.920	.424	68°	1.187	.927	.375	2.475
24°	.419	.407	.914	.445	69°	1.204	.934	.358	2.605
25°	.436	.423	.906	.466	70°	1.222	.940	.342	2.747
26°	.454	.438	.899	.488	71°	1.239	.946	.326	2.904
27°	.471	.454	.891	.510	72°	1.257	.951	.309	3.078
28°	.489	.470	.883	.532	73°	1.274	.956	.292	3.271
29°	.506	.485	.875	.554	74°	1.292	.961	.276	3.487
30°	.524	.500	.866	.577	75°	1.309	.966	.259	3.732
31°	.541	.515	.857	.601	76°	1.326	.970	.242	4.011
32°	.558	.530	.848	.625	77°	1.344	.974	.225	4.331
33°	.576	.545	.839	.649	78°	1.361	.978	.208	4.705
34°	.593	.559	.829	.674	79°	1.379	.982	.191	5.145
35°	.611	.574	.819	.700	80°	1.396	.985	.174	5.671
36°	.628	.588	.809	.726	81°	1.414	.988	.156	6.314
37°	.646	.602	.799	.754	82°	1.431	.990	.139	7.115
38°	.663	.616	.788	.781	83°	1.449	.992	.122	8.144
39°	.681	.629	.777	.810	84°	1.466	.994	.104	9.514
40°	.698	.643	.766	.839	85°	1.484	.996	.087	11.43
41°	.716	.656	.755	.869	86°	1.501	.998	.070	14.30
42°	.733	.669	.743	.900	87°	1.518	.999	.052	19.08
43°	.750	.682	.731	.933	88°	1.536	.999	.035	28.64
44°	.768	.695	.719	.966	89°	1.553	1.000	.017	57.29
45°	.785	.707	.707	1.000	90°	1.571	1.000	.000	∞

EXPERIMENT A-1
Worksheets

Name ________________________________

Table I

θ_1	θ_r	θ_2
0°		
5°		
10°		
15°		
20°		
25°		
30°		
35°		
40°		
45°		
50°		
55°		
60°		
65°		
70°		
75°		
80°		
85°		

1. ________________________________

2. ________________________________

3. ________________________________

4. ________________________________

5. ________________________________

6. ________________________________

7. ________________________________

8. ________________X________________

9. D = ______________in.
D = ______________cm
D = ______________mm

10. ____
____
____
____

11. ____
____
____
____
____

____
____

12. Magnification = ________________

13. ____
____
____
____
____
____
____
____

COMPUTATION SHEET

EXPERIMENT A-2
Work Sheets

Name ______________________

1. lens 1: f = __________ cm
   lens 2: f = __________ cm

2. __________ cm

3. ______________________

4. Objective, f_o = __________
   Eyepiece, f_e = __________

5. ______________________

6. Magnification __________

7. Objective, f_o = __________
   Eyepiece, f_e = __________

8. ______________________

9. Magnification __________

10. ______________________

11. ______________________

12. ______________________

13. ______________________

14. ______________________

15. ______________________

16. ______________________

COMPUTATION SHEET

EXPERIMENT B-1
Work Sheets

Name____________________________

1.

2.

3.

4.

$\sin \theta_1$	$\sin \theta_2$

5.

6. Slope = __________

7. __________ = __________

8.

9.

10.

EXPERIMENT B-2

Work Sheets

Name______________________________

1. Position of Lens = ____________

2. Position of one focal plane = ____________

3. Position of other focal plane = ____________

4. Angular size of distant object = ____________radians.

5. H_i = ____________cm

6. $f = H_i/\alpha$ = ______________
   = ______________cm

7.

Object Position	Image Position

8.

x (Object Distance)	x' (Image Distance)

9.

xx'	f^2

COMPUTATION SHEET

EXPERIMENT C-1
Work Sheets

Name______________________________

1.

Lens	Measured focal length f_o	Estimate of error in focal length measurement Δf_o
Lens A		
Lens B		
Lens C		
Lens D		
Lens E		

2. f_e = ______________

3. Magnifications predicted by theory

M	f_o

4. Plot a graph of the data in step 3.

6. Experimentally Observed Magnifications

Lens	M	ΔM	f_o	Δf_o
A				
B				
C				
D				
E				

7. Slope of experimental line
= ____________
Maximum slope = ______________
Minimum slope = ______________
Slope =

$$\_\_\_\_\_\_\_\_ \pm \frac{\text{Max slope} - \text{Min slope}}{2}$$

= ____________ ± ____________

8. ______________________________________
____
____

EXPERIMENT C-2
Work Sheets

Name______________________________

1. M = ______________

2.

D (cm)	ΔD	d (cm)	Δd

3.

$M = D/d$	$\frac{\Delta D}{D} \times 100$	$\frac{\Delta d}{d} \times 100$	Sum (percentage error in D/d)

4. Average value of M = ______________
   Average percentage error = ______________
   Error estimate in $M = D/d$
   = Average percentage error × average of D/d

Experimentally observed value of
M = ____________ ± ____________

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